Moment of Inertia and Angular Momentum

Vocabulary

**Moment of Inertia:** The resistance of an object to changes in its rotational motion.

The equation for the moment of inertia varies depending upon the shape of the rotating object. For an object rotating around an axis at a distance \( r \),

\[
\text{moment of inertia} = \text{(mass)}(\text{radius})^2 \quad \text{or} \quad I = mr^2
\]

The SI unit for moment of inertia is the **kilogram · meter squared** (kg · m²).

Other moments of inertia can be found in your textbook, and are summarized as follows.

- hoop rotating about its center: \( I = mr^2 \)
- hoop rotating about its diameter: \( I = \left(\frac{1}{2}\right)mr^2 \)
- solid cylinder: \( I = \left(\frac{1}{2}\right)mr^2 \)
- stick rotating about its center of gravity: \( I = \left(\frac{1}{12}\right)mt^2 \)
- stick rotating about its end: \( I = \left(\frac{1}{3}\right)mt^2 \)
- solid sphere rotating about its center of gravity: \( I = \left(\frac{2}{5}\right)mr^2 \)

Newton’s first law says that inertia is the tendency of an object to stay at rest or remain in motion in a straight line with a constant speed unless acted upon by an unbalanced force. Similarly, an object that is rotating tends to continue spinning at a constant rate unless an unbalanced force acts to alter that rotation. This is called the rotational inertia.

Think of moment of inertia as being the rotational equivalent of the term “mass.” Just as inertia is greater for a greater mass, rotational inertia is greater for a greater moment of inertia.

Vocabulary

**Angular Momentum:** The measure of how difficult it is to stop a rotating object.

\[
\text{angular momentum} = \text{(mass)}(\text{velocity})(\text{radius}) \quad \text{or} \quad L = mvr
\]

The SI unit for angular momentum is the **kilogram · meter squared per second** (kg · m²/s).

Think of angular momentum as being the rotational equivalent of linear momentum. Just as linear momentum is the product of the mass and the velocity, angular momentum is the product of the mass and the velocity for an object rotating at a distance \( r \) from the axis.
Momentum is conserved when no outside forces are acting. Similarly, angular momentum is conserved when no outside torques are acting. A spinning ice skater has angular momentum. When the skater pulls her arms in (decreasing her radius of spin), she spins faster (increasing her velocity). Doing so conserves her angular momentum.

**Solved Examples**

**Example 7:** On a game show, a contestant spins a 15.0-kg wheel that has a radius of 1.40 m. What is the moment of inertia of this disk-shaped wheel?

**Solution:** A disk is a thin cylinder, so the moment of inertia of a disk is the same as that of a cylinder.

\[
\text{Given: } m = 15.0 \text{ kg} \\
\text{Unknown: } I = ? \\
\text{Original equation: } I = \left(\frac{1}{2}\right)mr^2
\]

**Solve:**

\[
I = \left(\frac{1}{2}\right)(15.0 \text{ kg})(1.40 \text{ m})^2 = 14.7 \text{ kg} \cdot \text{m}^2
\]

**Example 8:** Trish is twirling her 0.60-m majorette’s baton that has a mass of 0.40 kg. What is the moment of inertia of the baton as it spins about its center of gravity?

**Solution:**

\[
\text{Given: } m = 0.40 \text{ kg} \\
\ell = 0.60 \text{ m} \\
\text{Unknown: } I = ? \\
\text{Original equation: } I = \left(\frac{1}{2}\right)mt^2
\]

**Solve:**

\[
I = \left(\frac{1}{2}\right)mt^2 = \left(\frac{1}{2}\right)(0.40 \text{ kg})(0.60 \text{ m})^2 = 0.072 \text{ kg} \cdot \text{m}^2
\]

**Example 9:** At Wellesley College in Massachusetts there is a favorite tradition called hoop rolling. In their caps and gowns, seniors roll wooden hoops in a race in which the winner is said to get her wish. Hilary rolls her 0.2-kg hoop across the finish line. The moment of inertia of the hoop is 0.032 kg·m². What is the radius of the hoop?

**Solution:**

\[
\text{Given: } m = 0.2 \text{ kg} \\
I = 0.032 \text{ kg} \cdot \text{m}^2 \\
\text{Unknown: } r = ? \\
\text{Original equation: } I = mr^2
\]

**Solve:**

\[
r = \sqrt{\frac{I}{m}} = \sqrt{\frac{0.032 \text{ kg} \cdot \text{m}^2}{0.2 \text{ kg}}} = \sqrt{0.16 \text{ m}^2} = 0.4 \text{ m}
\]

**Example 10:** Jupiter orbits the sun with a speed of 2079 m/s at an average distance of 71,398,000 m. If Jupiter has a mass of 1.90 \times 10^{27} \text{ kg}, what is its angular momentum as it orbits?
Given: \( m = 1.90 \times 10^{27} \text{ kg} \)  
\( v = 2079 \text{ m/s} \)  
\( r = 71,398,000 \text{ m} \)

Unknown: \( L = ? \)

Original equation: \( L = mvr \)

Solve: \( L = mvr = (1.90 \times 10^{27} \text{ kg})(2079 \text{ m/s})(71,398,000 \text{ m}) = 2.82 \times 10^{38} \text{ kg}\cdot\text{m}^2/\text{s} \)

**Practice Exercises**

**Exercise 18:** Veanna is in Las Vegas waiting for her number to be called at the roulette wheel, a large 3.0-kg disk of radius 0.60 m. What is the moment of inertia of the wheel?

Answer: ______________

**Exercise 19:** Earth has a mass of \( 5.98 \times 10^{24} \text{ kg} \) and a radius of \( 6.38 \times 10^6 \text{ m} \). What is the moment of inertia of Earth as it turns on its axis?

Answer: ______________

**Exercise 20:** Olga, the 50.0-kg gymnast, swings her 1.6-m-long body around a bar by her outstretched arms. a) What is Olga’s moment of inertia? b) If Olga were to pull in her legs, thereby cutting her body length in half, how would this change her moment of inertia? (Assume her mass is evenly distributed all along her body.)

Answer: a. ______________

Answer: b. ______________
Exercise 21: Priya removes her 0.012-kg, 0.60-cm-diameter wedding band and spins it on the coffee table on its edge. What is the moment of inertia of the ring?

Answer: ________________

Exercise 22: Hickory dickory dock, the 20.0-g mouse ran up the clock, and took turns riding on the 0.20-m-long second hand, the 0.20-m-long minute hand, and the 0.10-m-long hour hand. What was the angular momentum of the mouse on each of the three hands?

Answer (second hand): ________________

Answer (minute hand): ________________

Answer (hour hand): ________________

Exercise 23: In a physics experiment, Ingrid, the ice skater, spins around in the rink at 1.2 m/s with each of her arms stretched out 0.70 m from the center of her body. In each hand she holds a 1.0-kg mass. If angular momentum is conserved, how fast will Ingrid begin to spin if she pulls her arms to a position 0.15 m from the center of her body?

Answer: ________________

Additional Exercises

A-1: In the Biblical tale of David and Goliath, the giant is slain when David hits him with a rock that he has spun around overhead in a sling. If the rock is spun with a frequency of 100 revolutions per minute, what is the rock’s period?
A-2: Ashton the ant is crawling on the still blade of a ceiling fan when the fan is turned on, causing Ashton to go for a ride. If Ashton sits on the fan blade at a distance of 0.80 m from the center of the fan and turns with a frequency of 1.2 Hz, a) how fast does Ashton spin? b) If Ashton slips off the spinning fan, describe the path he will take.

A-3: In “Rumpelstiltskin,” the miller's daughter is spinning straw into gold on a spinning wheel that turns at a speed of 7.5 m/s, making one revolution every 0.50 s. How long is a strand of gold that makes one complete turn around the wheel?

A-4: A 3.20-kg hawk circles overhead in search of prey. a) If the hawk circles once every 10.0 s in a circle 12.0 m in radius, what is the linear speed of the hawk? b) What centripetal force allows him to remain in a circle? c) What is providing the centripetal force?

A-5: Sasha’s favorite ride at the fair is the Ferris wheel that has a radius of 7.0 m. a) If the ride takes 20.0 s to make one full revolution, what is the linear speed of the wheel? b) What centripetal force will the ride exert on Sasha’s 50.0-kg body? c) Does Sasha feel as if she is being pulled in or out by the ride? d) Explain the difference between what she feels and what is really happening at the top and bottom of the wheel.

A-6: In order for Sasha (in A-5) to feel weightless at the top of the ride, a) at what linear speed must the Ferris wheel turn? b) At this speed, how much will she appear to weigh at the bottom of the Ferris wheel?

A-7: Earth orbits the sun approximately once every 365.25 days at an average distance of about $1.5 \times 10^{11}$ m. The mass of Earth is $5.98 \times 10^{24}$ kg. a) What is the centripetal acceleration of Earth? b) What is the centripetal force of the sun on Earth? c) What is the centripetal force of Earth on the sun? d) If this force exists between the sun and Earth, does this mean that Earth is “falling into” the sun? Explain.

A-8: Most doorknobs are placed on the side of the door opposite the hinges instead of in the center of the door. a) Why is this so? b) If a torque of 1.2 N·m is required to open a door, how much force must be exerted on a doorknob 0.76 m from the hinges compared to a doorknob in the middle of the door, 0.38 m from the hinges?

A-9: Priscilla is working out in the gym with a 2.00-kg mass that she holds in one hand and gradually lifts up and down. a) Will Priscilla find it easier to lift the mass if she pivots her arm at the shoulder or at the elbow? b) If Priscilla’s arm is 0.60 m long from her shoulder to her palm and 0.28 m long from her elbow to her palm, how much torque must she produce in each case to lift the weight?
A-10: Leif and Paige are rearranging the heights of their movable bookshelves; they remove one of the 2.00-kg, 0.60-m-long shelves by holding opposite ends. A 5.00-kg stack of books is piled up on the shelf 0.20 m from Leif. How much force must Leif and Paige each exert to hold the shelf level?

A-11: Brewster hits a 0.30-kg pool ball across the pool table and sinks it in the side pocket. If the pool ball has a radius of 3.5 cm, what is its moment of inertia as it rolls?

A-12: Rocky, a raccoon, squeezes into a 0.60-m-diameter cylindrical trash can to find a late-night snack. However, the can tips over and begins to roll. If Rocky and the can have a combined mass of 40.0 kg, what is the moment of inertia of the system?

A-13: Mieko sharpens a knife on a grinding wheel whose angular momentum is 27 kg·m²/s. The 5.0-kg wheel has a radius of 0.30 m. What is the linear speed of the wheel?

Challenge Problems for Further Study

B-1: The “Bake-a-Lite” Cake Company truck is on its way to deliver a birthday cake for the MacKenzie party when it rounds a curve of radius 20.0 m at a speed of 12 m/s. What coefficient of friction is needed between the cake pan and the truck in order to keep the pan from slipping?

B-2: On his way home from the office, Steven’s car rounds an unbanked curve that has a radius of 100 m. If the coefficient of friction between the tires and the road is 0.40, what is the fastest speed at which the car can round this curve without risking an accident?

B-3: Pretending to be Tarzan, 50.0-kg Zach swings from the end of a 5.0-m-long rope attached to a tree branch. The tree branch will break if subjected to a force greater than 750 N. What is the maximum speed with which Zach can swing in order to avoid breaking the branch?

B-4: Hanging in front of the office of Lewis Skeirik, D.D.M., is a sign that weighs 120 N and is suspended at the end of a 0.80-m-long support beam that weighs 10.0 N, as shown. What is the tension in a supporting wire that holds the sign at an angle of 20.0°?