Symmetrical Experiments to Test the Clock Paradox

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Abstract

The kinetic and dynamic theories to resolve the clock paradox of the special theory of relativity are critically reviewed. It has been shown that the argument based on asymmetry is invalid and the signal-counting scheme is not the correct explanation of differential aging. The clock paradox is restated in the symmetrical twin brothers (2-J) and quadruplet brothers (4-J) experiments. These experiments show that the clock paradox is inherent in the Lorentz transformation, or any transformation in which time is dependent on space coordinates and velocity. The 4-J experiment also shows that the relativity of simultaneity may lead to a paradox of occurrence. The around-the-world atomic clock experiment and the lifetime measurements of high energy particles are scrutinized theoretically and methodologically.

1. Introduction

The relativistic clock paradox was first mentioned by Einstein [1], and later discussed in more detail by Langevin [2], Laue [3] Lorentz [4] and Pauli [5]. The issue has attracted great attention of the scientific community ever since [6-26]. Indeed, nothing is more important and fundamental than the concept of space and time that defines our whole paradigm of physics, and no logical flaw on this matter should be allowed to escape our scrutiny. The clock paradox is very powerful because it presents a direct challenge to Einstein's prediction of the differential aging effect [7], a prediction vitally important to the validity of the theory of relativity. A champion among the seasoned scientists who believed that Einstein made a mistake was Herbert Dingle, a past President of the Royal Astronomical Society in the 50's [8-13]. The most persistent opponent of Dingle was William McCrea, also a past President of the Royal Astronomical Society [14-18]. Their debate was as hot as it was tiring, and they were asked to end their argument with concluding remarks. The heat of discussion has subsided since then. The issue, however, has by no means been settled. It may take a million experiments to build our confidence in a theory, but it takes only one paradox to forfeit it. It is in this spirit that the issue of clock paradox is revisited. The kinetic and dynamic theories to resolve the paradox are critically reviewed in the next section. The symmetrical twin and quadruplet experiments are presented in section 3 to remove any arguments based on asymmetry. The related issues of simultaneity and causality are discussed in section 4. The experimental verification of the differential aging by the around-the-world atomic clock is discussed in section 5, and the lifetime measurements of high energy particles are discussed in section 6.

2. The Clock Paradox

The well known clock paradox can be briefly stated as follows: The twin brothers, Jack and John, endeavor to carry out a space travel experiment to test the time dilation of special relativity. John the astronaut flies a shuttle into space with a speed comparable to the speed of light, and returns home after a number of years, while Jack stays home on the Earth. They synchronize their clocks before dear John takes off. According to the theory of relativity, time runs slow on the fast moving space shuttle and John should look younger than his twin brother Jack at the time of their reunion. On the other hand, Jack can be considered to be moving in the opposite direction with exactly the same speed with respect to John simply because the motion is relative, and Jack should look younger than John at the time of their reunion!

The efforts to resolve the above clock paradox fall, by and large, into two categories: the one that ignores the time dilaton of the accelerating period, which we shall refer to as the kinetic school, and the one that stakes the whole business on the effect of acceleration, which we shall refer to as the dynamic school. The two Genies are empowered by the same magic lamp: the asymmetry of the experiment, and try to fulfill the same wish: John comes back younger than his twin brother by a factor of γ. It is argued that John has to accelerate and decelerate to return and Jack gets to sit on Earth and twist his thumbs, the situations for the twins are asymmetrical, and therefore no paradox. It turns out, magically, that the poor hard working brother John, who has to suffer all the ordeal of mechanical shock, long term fatigue and loneliness, lives happier and stays younger than his twin brother Jack. In the following we will examine the two schools of thought separately.

2.1. The kinetic theory to resolve the clock paradox.

According to the kinetic theory, the differential aging is due to the route dependence of the events in the space-time diagram. Fig.1a shows the space-time diagram of the twins. The world line of Jack is along the vertical time axis, running from the origin of the reference frame to a point t on the time axis. The outgoing and returning world lines of John in Jack's reference frame make the other two sides of a triangle defined by the departure, the turnaround and the reunion points O, C and F. The small curving sections of OA, BCD and EF are the acceleration curves. The route of events for the two brothers are different and the time accrued along the two routes are supposed to be different according to the special theory of relativity. Darwin [19] suggested a numerical example to show how the differential aging is possible based on the kinematics, ignoring the time dilation during the accelerating periods. Such an approximation is reasonable since the cruising part can be made arbitrarily long. It is, however, a mistake to say that the kinetic theory ignores the effect of acceleration altogether. As a matter of fact, the hidden effect of acceleration in the kinetic theory is almighty: it outlaws a symmetrical space-time diagram, Figure 1b. Such a moratorium actually forbids the inverse Lorentz transformation because the space-time diagram is nothing but the graphical representation of the algebraic Lorentz transformation and its inverse. In essence, the kinetic school demanded that the validity of the Lorentz transformation and its inverse depended on who brought the relative motion, and how. It violates the very spirit of relativity by assuming some absolute frame of reference. It forgets that even the acceleration is symmetrical for both twins: In the eyes of the flying brother, his brother on Earth is receding with the same acceleration.

A technique employed by the kinetic school to resolve the paradox is the light-signal-counting technique. It is asserted that the time can only be measured accurately by counting the light signals sent by other observers [19-26]. Such practice inevitably leads to the well-known wrong conclusion that the receding clock runs slow while the approaching clock runs fast, which is essentially a Doppler effect. But the relativistic time dilation depends on the speed, not the velocity; it is supposedly valid for any one-directional trip, not just for a round-trip. Namely, the amount of delay is the same as long as the speed is the same, regardless of which way the system travels. It is absolutely a transformational effect, i.e., it is a direct consequence of the Lorentz transformation. Unfortunately, this distinction between the relativistic time dilation and the Doppler effect is not noted and the signal counting scheme is widely adopted in various textbooks.

2.2. The dynamic theory to resolve the clock paradox.

The dynamic school, on the contrary, does generously allow a legitimate diagram of Figure 1b which leads to the time dilation of Jack's clock by the same factor γ :

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

where u is the speed of Jack and c is the speed of light. The time dilation during cruising, according to this school, is overcompensated by the effect of acceleration [27,28]. The gain due to acceleration is exactly twice as much as the loss due to the Lorentz transformation, no matter how long the cruising part of the journey might be compared to the acceleration period. So the result is the same: John is younger by a factor of γ. Tolman [27] gave an approximate calculation, which was gladly reproduced in many publications and textbooks, based on the principle of equivalence. His thread of thinking can be described with the aid of Figure 2. Since the motion is relative, we can consider that Jack travels away from John in the opposite direction. Jack departs John at the point O where they synchronize their clocks. He then accelerates away from John for a short while until he reaches the point A. He then cruises a long journey to a remote point B, then decelerates to a stop at point C, turning back to point B with the same acceleration, cruises back to A, and gets home after a short landing AO. All these motions are observed by John in his space shuttle.

The time intervals are calculated as usual when Jack is considered at rest:

$$\Delta t_A = \frac{\Delta t_B}{\sqrt{1 - u^2/c^2}} = \Delta t_B \left(1 + \frac{1}{2}\frac{u^2}{c^2} + \dots\right) \qquad (1)$$

where Δt_A and Δt_B are the time intervals from the departure to reunion recorded by the resting Jack and the flying John, respectively, and u is the speed of their relative velocity. Tolman did agree that, in the name of relativity, Jack can be considered moving and the space-time diagram Figure1b should be allowed. Jack's world lines are drawn in John's reference frame which appear symmetrical to Figure 1a. And yes, the inverse Lorentz transformation was allowed and Jack's clock loses by a factor of γ during the cruising part of the journey both ways:

$$\tau_A = \tau_B \sqrt{1 - u^2/c^2} = \tau_B \left(1 - \frac{1}{2}\frac{u^2}{c^2} + \ldots\right) \qquad (2)$$

But Jack's clock gains during the turnaround period to compensate for the loss during cruising. According to Tolman, the elapsed time registered by Jack's clock, Δt_A, and that registered by John's clock, Δt_B, consist of four parts:

$$= \tau_A + \tau_A' + \tau_A'' + \tau_A''' \qquad (3)$$

$$= \tau_B + \tau_B' + \tau_B'' + \tau_B''' \qquad (4)$$

where τ_A and τ_B are the round trip time measurements of the two clocks during cruising. τ_A', τ_A'', τ_A''' and τ_B', τ_B'', τ_B''' are the time measurements of the two clocks at the departure, turnaround and landing periods by the temporary introduction of an appropriate gravitational field.

Tolman ignored the time changes during the take-off and the landing on the grounds that the twins are spatially close and the time of the acceleration periods are short, i.e., it is assumed that $\tau_A' = \tau_B'$ and $\tau_A''' = \tau_B'''$. The turnaround part of the journey, however, is significant and so is the time gain in this period. Tolman introduced a gravitational field at the turnaround to produce the necessary reversal in the motion of John. The time gain was given by

$$'' = \tau_B'' (1 + \Delta\Psi / c^2) \qquad (5)$$

where $\Delta\Psi$ was the difference in gravitational potential at the turnaround:

$$\Delta\Psi = h\, g \qquad (6)$$

He then assumed that h was the total distance from A to B:

$$h = (1/2)\, u\, \tau_B \qquad (7)$$

$$g = 2\, u / \tau_B'' \qquad (8)$$

Substituting Eqs. (6)-(8) into Eq. (5) yields

$$'' - \tau_B'' = \tau_B (u^2 / c^2) = 2 (\tau_B - \tau_A) \qquad (9)$$

Eqs. (2)-(4) and (9) together give same result as Eq. (1). The paradox seems to be solved.

An apparent flaw in the above calculation is that one can not assume $\Delta\Psi = h\, g$ unless Jack and John are separated by a distance h all the time with a potential difference $\Delta\Psi$. Since such distance is achieved gradually with constant velocity, h in Eq. (7) should be multiplied by a factor of one half when entering into the time dilation calculation, Eqs. (5) and (6). This factor of one half is vitally important for Tolman's argument.

Another problem with Tolman's technique is that it depends crucially on the approximation Eq. (2), as properly pointed out by Mendel Sachs [29], Tolman's resolution of the clock paradox will fail if the velocity is high and the higher order terms in Eq. (2) have to be kept. Any theory attempting to resolve the paradox has to be valid not only for low velocity, but also for the relativistic velocity.

And there is the magic of turning on and off the gravitational field. The matter of fact is, no one in our physical world can turn any gravitational field on and off at will. The issue here is not just about a technical problem, but about a fundamental principle. To carry out the impossible task of turning on and off the gravitational field, one needs nothing less than Maxwell's demon who can defeat the second law of thermodynamics by turning on and off a valve on the partition at the convenient time. As a matter of fact, the "technical" problem of turning on and off a gravitational field is not any easier than the "technical" problem of making a round trip without acceleration, a key argument that both schools of thought use to undergird the assertion of differential aging.

Nevertheless, we can imagine, in the spirit of the equivalence principle, that Jack is accelerated and decelerated by a certain gravitational field, instead of fuel propellant, to realize the same trip. We can visualize the picture like a skier skiing back and forth between two hills. The potential of such a gravitational field is plotted as a function of distance in Figure 3. The points O, A, B and C are the same as designated in Figure 2. The arrows on the potential curve indicate the direction of the outgoing trip. The returning trip follows the same potential curve with the direction reversed. The points A and B are equipotential, so are points O and C. The diagram shows that the distance h in Eq. (6) should be approximately equal to one half of the distance BC (because Jack does not jump from the point B to the point C abruptly). Surely h is not equal to AB. Eqs. (7) and (9) are evidently invalid. The invalid assumption of h = AB leads to the absurdity of Equation (9) which says that the time gain during the turnaround does not depend on the time of turnaround, rather, it is proportional to the cruising time!

We can easily calculate the potential difference between the points B and C. Instead of Eq. (6), we should correctly have $\Delta\Psi$ = BC g. But BC = (1/2) g $(0.5\ \tau_B'')^2$ and g = 2 u / τ_B''. We immediately have $\Delta\Psi = 0.5\ u^2$, which is independent of the cruising time τ_B. The mistake contained in Eq. (9) also violates Einstein's time hypothesis which states that the time rate is not affected by the acceleration.

As we can see from Figure 3, there is no potential difference between points O and C, or between points A and B. The potential difference exists in the regions OA and BC only, and the potential differences in the take-off and landing periods should be just as significant as it is in the turnaround period. There is no reason to credit the turnaround period more than the take-off and the landing period, as far as the gravitational effect is concerned.

According to the general theory of relativity, time runs slow at low gravitational potential. Figure 3 shows that Jack stays at the low gravitational potential most of the time, and the gravitational potential should help Jack's clock to lose time. The Lorentz effect and the effect of gravitational field introduced by Tolman should be constructive instead of destructive. That is, $\Delta\Psi$ in Eq. (6) should be negative. Introduction of a gravitational field does not resolve the paradox, but exacerbates it.

Finally, Tolman failed to show why, in the spirit of relativity, the principle of equivalence does not come into play when Jack's reference frame is taken as the rest frame. Isn't John moving away from Jack with acceleration of equal magnitude?

A fundamental question should be asked: Does the principle of equivalence demand that any acceleration effect has to be replaced by a "portable", "local" gravitational field that can be conveniently turned on and off, never mind how? We know that there are four fundamental forces and they can not be replaced by each other. On what bases should the gravitational interaction presumptuously assume an almighty supremacy over the other fundamental interactions?

The claim that the twin paradox can not be resolved without resorting to the theory of general relativity does nothing but to confess the impotence of the special theory of relativity. It has been sharply criticized and dismissed by the kinetic school. Builder [23, 24] commented that the "relative retardation is predicted by the restricted theory of relativity, taken together with the assumption that the 'rate' of a clock depends only on its velocity and not on its acceleration.", and that "the general theory of relativity can add nothing significant to the analysis" of the paradox, "the application of the principle of equivalence is essentially trivial; in effect, Einstein and Tolman evaded the real logical issue raised by the contradictory predictions by denying the applicability of the restricted theory and then utilizing, by means of the principle of equivalence, results obtained from it. This tortuous procedure succeeded in evading the paradox rather than in resolving it; it would obviously be quite invalid were the restricted theory indeed inapplicable to the problem." The reason that Einstein and Tolman were not quite satisfied with the kinetic explanation was probably due to their full conviction of the principle of relativity which states that all motions are relative, and no excuse as to how and who brings about such relative motion should prevent either of the twins from being taken as the rest reference frame. (Isn't that what the very word "reference" actually means?) Einstein and Tolman might have evaded or denied the paradox, but the kinetic school has evaded and denied the principle of relativity. Einstein might not know what his disciples would do to defend his theory, but one thing he did know, we can trust, was what he meant in his theory of relativity. He meant that all motions are relative and any observer can be considered as at the rest, regardless of who caused the motion, and for every Lorentz transformation there should exist an inverse transformation, and both the transformation and the inverse transformation can be represented by corresponding space-time diagrams.

In his book "Principles of Relativity Physics" [30], Anderson maintained "we have assumed that the laws of special relativity hold throughout the space-time region in question, there is no reason for bringing general relativity into the argument." He insisted that "of course all motion is not relative in special relativity. Acceleration is just as absolute in this description as in Newtonian mechanics", and therefore, Figure 1b. is not allowed. But he did allow the counting of Jack's age by John during the cruising time referring to Figure 1a. The events B and B' are simultaneous according to John, so are the events D and D'. Jack's aging from the event O to B' plus that from D' to F, in the eyes of John, is calculated according to Lorentz transformation, and it is a factor of γ younger than John's round trip age, which is the same as given by Tolman in Eq. (2). However, Jack has to live a long lone while from the events B' to D', waiting for John to turnaround which supposedly takes very little time. One week in the Kingdom of Heaven equals ten thousand years on the Earth! According to Anderson, Jack's aging during John's short turnaround needs to be just right, i.e., Jack has to age an amount $\tau_B(u^2/c^2)$ during John's quick turnaround, to resolve the paradox and save the theory of relativity. The results is of course the same as Eq.(9). Anderson harvested the full basket of Tolman's fruit by criticizing his unprofessional cultivation.

In the following section, we will formulate two symmetrical experiments to show that the asymmetry argument is not able to resolve the twin paradox.

3. The symmetrical twin and quadruplet experiments

The analysis in section 2 has shown clearly that the clock paradox is a logical one, and the asymmetry has nothing to do with the essence of the issue. To convince those who are incapable of expelling the phantom of asymmetry and grasp the essence of the clock paradox, we here entertain them with two completely symmetrical experiments: one employing twin brothers, which shall be referred to as the 2-J experiment, and one employing quadruplet brothers, which shall be referred to as the 4-J experiment.

3.1 The 2-J Experiment

In this experiment we let both twins start their journey from a space station far from any heavenly bodies, so that the whole experiment can be carried out in free space.

As depicted in Figure 4, Jack and John are equipped with identical "twin" shuttles and "twin" clocks synchronized at the departure point O. The twins travel in opposite directions along the same straight line. Other than the direction, their accelerating and cruising processes are pre-programmed to be identical as measured by their own speedometers, clocks and accelerometers. Thus, they start their journeys with the same preset acceleration for the same time period t_1 as read from their own clocks to reach a relativistic speed v at the points A and A', and then cruise for a preset long period t_2 to the points B and B'. They start turning back with the same deceleration for a time $2t_1$ to return back to points B and B' with their velocities reversed and cruise back home for a time t_2. The deceleration for the landing is also symmetrical. Such a symmetrical experiment should guarantee that the twins land at the original departure point O simultaneously at the same time as registered by both clocks (equal aging). John's world line in Jack's reference system is represented in Fig. 1a Their relative cruising velocity is given by special relativity to be $u = 2v/(1+v^2/c^2)$.

The perfectly symmetrical experiment allows no filibustering argument based on asymmetry. Both Fig 1a and Fig. 1b are equally legitimate, with Fig. 1b showing the symmetrical world line of Jack in John's reference frame. The special theory of relativity predicts that if Jack is considered to be at rest, John should be younger than Jack at the time of reunion. If, however, John is considered to be at rest, then Jack should be younger than John. The theory of relativity gives two contradictory predictions, depending on who is mentally taken to be the rest reference system. But we know both predictions are wrong since, as reasoned above, the symmetric arrangement of the experiment dictates that the twins must have aged the same by the reunion.

Can the dynamic theory come to our rescue? Could the acceleration cause a compensating effect to cancel the Lorentz transformational time dilation so that the twin brothers age the same by the reunion? The answer is no. To examine this, let us assume the total differential aging Δt consists of two parts, $\Delta\tau_u$ and $\Delta\tau_a$:

$$\Delta t = \Delta\tau_u + \Delta\tau_a$$

where $\Delta\tau_u$ is the differential aging resulted from Lorentz transformation during the cruising periods, and $\Delta\tau_a$ is the differential aging during the accelerating periods. Since the two brothers must have the same age after taking symmetrical journeys, we must have

$$\Delta\tau_u + \Delta\tau_a = 0 \qquad (10)$$

Eq. (10) requires that $\Delta\tau_a$ and $\Delta\tau_u$ must be opposite in sign and equal in magnitude. But this is impossible. First, the two components can not be equal in magnitude since $\Delta\tau_u$ is proportional to the arbitrarily long cruise time, while $\Delta\tau_a$ should depend solely on the mathematical structure of the turning curve preset by the acceleration program. One is therefore forced to commit the same sins contained in Eq. (9), unless both $\Delta\tau_u$ and $\Delta\tau_a$ vanish. Second, the two components can not be opposite in sign because it violates Einstein's clock hypothesis which states that the instantaneous rate of a clock depends only on its instantaneous speed but not on its acceleration. The time dilation of an accelerating system is assumed to be the same as that of a co-moving system with the same instantaneous velocity u. The time along the relevant part of the world line with acceleration is given by the integral

$$t' = \int_0^t \gamma \, dt \qquad (11)$$

where t is the proper time of the accelerating traveler, and t' is the time measured by the observer. This hypothesis has been used by Einstein himself and others [31]. Eq. (11) shows that t' is always greater than t and the time dilation during the acceleration period can not possibly be negative. Moreover, it shows that the time dilation is solely determined by the integrand γ, a certain function independent of the past history.

3.2 The 4-J Experiment

We can formulate another experiment that keeps acceleration entirely out of the picture. We now enlist quadruplets, Jack, Jim, John and Joe, with four identical clocks and two identical long space shuttles. Jack and Jim shall ride on one shuttle, while John and Joe on the other. Jack and John shall sit in the front cockpits of their shuttles, while Jim an Joe in the rear cabins. Each of the quadruplet brothers shall carry a clock, and the two clocks in each shuttle are spaced at the distance L. All four clocks are synchronized at the moment of departure. The two teams shall take the same preprogrammed symmetrical travel and return home, as described above in the 2-J experiment, except for the landing portion. When the two teams come back home, they do not reduce their speed but continue cruising with the same speed and pass each other. At the moment when the two pilots Jack and John meet, they synchronize all four clocks. The synchronization of two clocks within the same inertial system is always allowed by the theory of relativity. As a matter of fact, they may not even need to do any physical synchronization, but merely check their clock readings, which are expected to be the same since all four clocks were synchronized at departure, and the trip is designed symmetrically. At the moment when Jack meets John in flight, the four brothers must have the same age due to symmetry. See Fig. 5.

After a certain amount of time the two brothers sitting at the rear, Jim and Joe, will meet. At this moment Jim's clock reads t' and Joe's reads t. The symmetry dictates that their clocks read the same time from their own clocks in their own reference frames. Namely,

$$t = t' = T \qquad (12)$$

which is to say that Jim and Joe must have aged the same T. On the other hand, Jack and John must age the same T because they must age at the same rate as their crew brothers do. But if Joe tries to calculate and compare their times according to the theory of special relativity, he should have

$$t = \gamma\, t' \qquad (13)$$

Likewise, Jim shall insist that

$$t' = \gamma\, t \qquad (14)$$

The results (12), (13) and (14) contradict each other and manifest the same clock paradox.

4. Simultaneity and causality

According to the theory of relativity, two events occurring simultaneously in one reference frame are in general not simultaneous to the observers in another moving frame. The non-unique simultaneity may lead to the reversed order of cause and consequence as observed in the different frames. The differential simultaneity and reversed causality are usually explained by saying that the signals sent simultaneously from different places may reach the observer in one reference frame simultaneously, but they do not in general reach an observer in another moving frame simultaneously. Such a hand-waving explanation does a pretty good job in quenching the curiosity of students, but it does not touch the real issue. If simultaneity means that the observer must detect the signals at the same time, then we can not even speak of any simultaneity at all even within the same inertial system! Simultaneous events reach the observer in the same system simultaneously only when these events take place on a circle with the observer at its center. But we know things can happen simultaneously even when they are not on the circle. As a matter of fact, all the events on any line parallel to the x axis in a space-time diagram are simultaneous to observers anywhere in that reference frame, but the light signals of these events will not reach any observer simultaneously. The relativistic simultaneity of events is entirely different from the difference in time of arrival. The 4-J experiment allows us to explore the consequences of the non-unique simultaneity of special relativity.

Referring to Fig. (6), let us examine the event E_A when pilot John meets passenger Jim, and the event E_B when pilot Jack meets passenger Joe. Classically, these two events should take place simultaneously because the two space shuttles have identical length. Relativistically, however, the two events are not simultaneous due to Lorentz contraction. To John and Joe, Jack and Jim's shuttle is shorter and John should meet Jim before Joe meets Jack, namely, E_A should take place before E_B. Since the motion is relative, Jack and Jim should expect E_B taking place before E_A. This result is considered "paradox of simultaneity and causality" by Newtonists, and "relativity of simultaneity and causality" by Einsteinists.

To settle the argument, some detailed calculations are in order. At the event E_A, John's coordinates in his own system are $(0,t_1)$, and Jim's coordinates in Jim's system are $(-L,t_2')$. Likewise, at the event E_B, Joe's coordinates are (L,t_2) and Jack's are $(0,t_1')$, as measured in their respective reference systems. These coordinates are related by Lorentz transformation:

$$0 = \gamma (-L + u\, t_2') \tag{15}$$

$$t_1 = \gamma (-Lu/c^2 + t_2') \tag{16}$$

$$L = \gamma\, u\, t_1' \tag{17}$$

$$t_2 = \gamma\, t_1' \tag{18}$$

We obtain

$$t_1 = L/(\gamma u) \tag{19a}$$

$$t_2 = L/u \tag{19b}$$

$$t_1' = L/(\gamma u) \tag{20a}$$

$$t_2' = L/u \tag{20b}$$

Eqs. (19a) and (19b) predict that John meets Jim before Joe meets Jack as observed by John and Joe, according to the theory of special relativity. On the other hand, Eqs. (20a) and (20b) predict that Joe meets Jack before John meets Jim as observed by Jack and Jim, according to the same theory of special relativity. This result is not considered paradoxical by the relativists, and it is explained by "Lorentz contraction" which states that the length of a moving object becomes shorter by a factor of γ. To John and Joe, Jack and Jim's shuttle is moving and contracted to a shorter length, therefore, when John meets Jim , Joe should be on the right side of Jack, waiting to meet Jack. See Figure 6.

But John is too good a pilot to forget to double check things. He wants to make sure that when E_A takes place Jack is on the left side of Joe, i.e., Jack's coordinate x in John's system should be less than Joe's coordinate L:

$$x < L \tag{21a}$$

At the moment of E_A, Jim's clock reads t_2', and Jack's clock must read the same t_2' because they stay in the same reference frame. Jack's coordinates in the Jack-Jim frame are therefore $(0,t_2')$, where $t_2' = L/u$ (Eq. 20b). Jack's coordinates as transformed to John-Joe's reference frame should be

$$x = \gamma\,(0 + u\,t_2') = \gamma\,(u\,L/u) = \gamma\,L > L\ ! \tag{21b}$$

Whoops! Somehow Jack has sneaked through, relativistically, to the right side of Joe without meeting him! The paradox of simultaneity and causalilty manifests itself as a paradox of occurrence, as stated in the contradicting Eqs. (21a) and (21b).

5. The Around-the-World Atomic Clock Experiment

The symmetric experiment described above seems to be quite a challenge at the current stage of technology. But the power of logic is that we do not even have to actually carry out the experiments to see the paradoxical predictions by the same theory. However, many have claimed that they have tested the relativistic time dilation experimentally, it is therefore instructive to examine these experiments theoretically and methodologically. The most noteworthy of these experiments has been the much celebrated around-the-world atomic clocks experiment carried out by J.C. Hafele and Richard E. Keating in 1971 [32,33]. They flew four portable cesium clocks around the world during October 1971, once eastward and once westward, and reported that their results "provide an unambiguous resolution of the famous clock 'paradox' with macroscopic clocks". The theoretical calculation related to the experiment was given by Hafele in [34].

In his paper [29], Hafele used a non rotating Schwarzschild metric

$$ds^2 = (1+2\chi / c^2)\, c^2\, dt^2 - [dr^2 / (1+2\chi / c^2) + r^2\, (d\theta^2 + \sin^2\theta\, d\phi^2)] \tag{22}$$

where r, θ and ϕ are the spherical coordinates, and χ is the scalar potential [35]. Under slow speed approximation, Eq.(22) gives the proper time

$$d\tau = [\,1 + (\chi / c^2) - (\,u^2 / 2\,c^2\,)]\,dt \tag{23}$$

where dt is the coordinate time interval, or the proper time of a reference frame at rest with respect to the remote north star, and u is the coordinate velocity. The finite time interval recorded by a clock flying around the Earth can be obtained by integrating Eq. (23) from the start to the final stop. Following this approach, Hafele yielded the ratio of the proper time registered by a clock flying along the equator to that of a clock sitting on the Earth:

$$\Delta\tau / \Delta\tau_o = 1 + (g\ h / c^2) - (2\ R\ \Omega\ v + v^2) / 2\ c^2 \qquad (24)$$

where g, h, R, Ω and v are, respectively, the gravitational acceleration on Earth, the flying height of the clock from the surface of the Earth, the radius of the Earth, the angular speed of the Earth and the ground speed of the flying clock. The relative difference of the times recorded by the flying and the sitting clocks is then given by

$$= (\Delta\tau - \Delta\tau_o) / \Delta\tau_o = (g\ h / c^2) - (2\ R\ \Omega\ v + v^2) / 2\ c^2 \qquad (25)$$

As an anonymous referee has rightly suggested, that the second term in Eq. (25) is a purely special relativistic effect, in spite of the apparent use of general relativity. The first term is apparently related to the gravitational effect on time. If the clock is flying along a latitude instead of the equator, Eq. (25) should be modified as follows:

$$\delta = (\Delta\tau - \Delta\tau_o) / \Delta\tau_o = (g\ h / c^2) - (2\ R\ \Omega\ v \cos\lambda + v^2) / 2\ c^2 \qquad (26)$$

where λ is the latitude.

The kinematic second term in Eq. (25) is negative except when $- 2\ R\ \Omega < v < 0$, i.e., when the clock is flying westward with small velocity. The velocity of the commercial jet with which Hafele and Keating flew their clocks fell in this range. According to their report [32], the eastward trip lasted 65.4 hours with 41.2 hours in flight. The westward trip lasted 80.3 hours with 48.6 hours in flight. They reported a time loss of 59 +/- 10 nanoseconds for the eastward trip and a 273 +/- 7 nanoseconds gain during the westward trip, in reasonable accord with the theoretical values of 40 and 275 nanoseconds predicted by Eqs. (25) and (26) (with modifications to take care of the fact that the trips were not along a latitude and that the jets may have changed its speed on route).

This experiment has been triumphantly celebrated as an unambiguous empirical resolution of the clock paradox with macroscopic clocks. Hardly anyone, though, realizes that Eq. (25) offers a direct proof, not the resolution, of the clock paradox.

Thus, we arrange two jets carrying two synchronized identical clocks flying along the equator at the same height h, see Fig. 7. Clock A flies eastward with ground velocity v and clock B flies westward with ground velocity

$$v' = - (v + 2\ R\ \Omega). \qquad (27)$$

Making use of Eq. (25), we have

$$\delta = (\Delta\tau - \Delta\tau_o) / \Delta\tau_o = (g\ h / c^2) - (2\ R\ \Omega\ v + v^2) / 2\ c^2 \qquad (28)$$

$$\delta' = (\Delta\tau' - \Delta\tau_o) / \Delta\tau_o = (g\ h / c^2) - (2\ R\ \Omega\ v' + v'^2) / 2\ c^2$$
$$= (g\ h / c^2) - (2\ R\ \Omega\ v + v^2) / 2\ c^2 \qquad (29)$$

Comparing Eqs. (28) and (29) gives

$$\delta' = \delta \qquad (30)$$

Eq. (30) says that whenever condition (27) is satisfied, the two clocks flying in the opposite sense shall have the same time rate and register the same time (or age) when the two meet again. Specifically, if $v=0$, $v'= -2R\Omega$. It means that a clock flying westward with velocity $v'= -2R\Omega$ should have the same time rate as the one sitting on the Earth. This directly contradicts the relativistic prediction that clock B will lose if clock A is TAKEN AS the rest frame, and clock A will lose if clock B is TAKEN AS the rest frame -- a logical paradox. This experiment is a different version of the 2-J experiment described in section 3. Introduction of gravity and general theory of relativity does not change the logic of the clock paradox.

One might contend that none of the flying clocks can be taken as the rest frame. The rest frame has to be taken with respect to the North star. This argument actually demands an absolute coordinate system, a notion in direct conflict with the principle of relativity. We know the North star is moving, so is the Milky Way, and even the Universe. One would actually invalidate the whole theory of relativity by demanding an absolute reference frame.

It should be noted that Eqs. (23) to (26) represent the theory of relativity faithfully. If these equations lead to a paradoxical result, it is not Hafele's fault. The paradox is deeply rooted in the Lorentz transformation, and it is logical. As a matter of fact, Hafele has made a contribution to the discussion of the clock paradox by demonstrating that the time dilation during a trip with acceleration is easily amenable to calculation, and it is not proportional to the past history, as some hoped to happen in order to cancel the effect during an arbitrarily long cruise.

If no good experiment should support any paradoxical theory, then what might have gone wrong with the around-the-world atomic clock experiment? The authors of this experiment revealed some of the possible error sources [33]: a) The number of the measured values was too small for a good statistical analysis; b) There were possible errors in the rates and rate changes used in the piece wise extrapolations. A preliminary study of the effect of adjustments in the calculated rates on the residuals between the calculated and measured time traces showed obvious distortions when deviations greater than 0.4 nsec per hour were arbitrarily introduced; c) Temperature or pressure changes could cause individual random and unpredictable changes

[36]. d) The instability of the portable cesium clocks used in this experiment was about 1 μsec per day and it was unpredictable [33]. This instability for the three days trip (64 hours eastward and 81 hours westward) was then about 3 μsec, much greater than the predicted time difference of 275 nsec. It was therefore not reliable in principle to draw conclusions based on a single cesium clock experiment, as the authors admitted. In order to draw a conclusion, they employed four clocks and exercised some "proper accounting for these identified rate changes" to yield a time change in agreement with the expected relativistic time change within 23 nsec. Statistically, however, one should not expect an improvement of accuracy of more than a factor of two if four clocks instead of one were used, i.e., it is reasonable to expect the error due to instability reducing to one half, about 1.5 μsec, still large enough to swamp the expected signal of 0.275 μsec. One also fails to see how the reported error of 23 nsec was reached.

There were further possible causes for error that were not mentioned in [33]:

a) Since the four cesium clocks are independent, we might as well treat them as four independent experiments, and see how many of them supported the time dilation theory. Although no value of time dilation for the individual clocks was furnished in reference [33], one can use Fig. 1 of the reference to ascertain them. The plot shows that no clock supported the prediction, although statistically we should expect two thirds, or at least two out of four clocks, to fall within one standard deviation, which is reportedly 23 nsec.

b) The time differences were measured with an electronic time interval counter to the nearest nanosecond [33]. The electronic time interval counter apparently did not have the required precision and stability to detect the time difference of the cesium clock.

c) The experimental time difference $\Delta\tau$ was taken as the difference in the time rate changes of the clocks before and after each trip. These rates were determined by the linear fits to the average data for an interval of 25 hours immediately before and after each trip. The 25 hours period seemed to be rather arbitrary as one would naturally expect a period of 64 or 81 hours to be more appropriate. The sensitivity of the results to the length of period for determination of rate changes was not given. A good way of testing the reliability of the method employed would be to let the clocks sit in a laboratory for a few weeks, and see if a similar time dilation (about 0.3 μs) could be produced during a three day period by the same method of data reduction, without any clock being carried to a circumnavigating trip.

d) The trips were subjected to many take-offs and landings, the effect of which was completely ignored, although the effect of temperature or pressure was briefly mentioned. Other conditions such as the stability of the power supply, the effect of acceleration on the clock as well as to the peripherals etc. may all play a role. When a precision of 10^{-12} (275 nsec per 80 hours) is pursued, all these effects need to be quantitatively addressed.

6. Meson Lifetime measurements

Lifetime measurements of high energy particles have been claimed to be the microscopic evidence of the relativistic time dilation [37-44]. It is well known that the decay times of radioactive isotopes, such as carbon-14, uranium-238 and potassium-40, are used to date archeological objects and rocks. Hardly anyone realizes that radioactive clocks, like any other type of clocks, are not perfect. A radioactive isotope can be used as a clock only under certain conditions, one of the most important being that the radioactive samples are not subjected to any collision. This condition, however, is not satisfied in any of the meson decay experiments to test relativistic time dilation. It is instructive to scrutinize these experiments theoretically and methodologically.

6.1. Theoretical

The relative activity of a radioactive sample is an exponential function of decay time:

$$R(t) = A(t)/A_0 = \mathrm{Exp}[-t/\tau] \qquad (31)$$

where A(t) is the activity at time t, A_0 is the initial activity, R is the survival ratio, and τ is the decay time constant. In carbon dating experiments the decay time t is directly measured with a clock. For decay of high energy particles with lifetimes shorter than 10^{-3} s, however, direct time measurement is no longer possible. One has to measure the decay length, or stopping length in a certain material, instead of decay time. And as such, the particles must have high kinetic energy to penetrate through a measurable length. The spontaneous radioactive decay is then complicated by the collisional processes in the target, and is usually target specific. When a beam of energetic particles penetrates through a target medium with density ρ, the survival ratio is an exponential function of the distance x:

$$R(x) = A(x)/A_0 = \mathrm{Exp}[-Bx] = \mathrm{Exp}[-\rho\sigma x] \qquad (32)$$

where B is the spatial decay constant and σ is the total collisional cross section including all the reactions responsible for the decay of the particles. If we express the decay length x in terms of the velocity v of the particle and the flight time t, Eq. (32) can be written as

$$R(t) = A(t)/A_0 = \mathrm{Exp}[-\rho\sigma vt] = \mathrm{Exp}[-t/\tau] \qquad (33)$$

with

$$\tau = 1/(\rho\sigma v). \quad (34)$$

A tacit assumption x = v t is inserted in Eq. (33), and the velocity v in the target is usually taken as the initial velocity of the particles before entering the target. In reality, the trajectory of the particle is much more complicated than that of a motion with constant velocity. Not only the magnitude, but also the direction of the velocity will change. It is therefore rather unreliable and indirect to take the apparent decay length as a measure of the decay time. Such phenomenological treatment can serve some practical purpose in certain applications, but is very unreliable for the highly accurate lifetime measurements.

The cross sections are additive, i.e.,

$$\sigma = \Sigma\sigma_i \quad (35)$$

where σ_i's are the cross sections pertinent to the various interactions responsible for destruction of the particles. Equivalent to Eqs. (34) and (35) is

$$1/\tau = \Sigma(1/\tau_i) \quad (36)$$

where $\tau_i = 1/(\rho\sigma_i v)$ is the time constant of the corresponding channel. The cross section of a nuclear interaction depends on the nature of the interaction potential and the collision energy, or the relative velocity. Such dependence is not as simple as a relativistic Lorentz factor γ, and it is in general not even a monotonic function of the velocity [45-54]. Two causes are believed to be responsible for the velocity dependence of the cross sections: 1) The interaction potential might be velocity dependent; 2) The interaction time within the interaction region is inversely proportional to the velocity of the impacting particle. This usually leads to a lower cross section and longer apparent lifetime at the high velocity range. For the charged particles, one also needs to consider the technical problems related to the lifetime measurement at the low energies: the low detection efficiency and high rate of charge loss due to electrostatic fields. This tend to give a shorter apparent lifetime at the lower speed range.

What about the spontaneous decay in vacuum? Does it involve any collisions? The answer is positive for the high energy particles. We know the vacuum is not a void. Even the "ideal" vacuum is actually matter from which the pair production of electron and positron can take place, not to mention the difficulties to obtain an "ideal" vacuum. The best vacuum we can obtain has a pressure of about 10^{-11} torr, corresponding to a gas density of 3.5×10^5 molecules per cubic centimeter. The practical vacuum pressure of a high energy particle beam line should be in the neighborhood of 10^{-7} - 10^{-8} torr, with a residue particle density of about 10^9 molecules per cubic centimeter. It is therefore not safe to say that the "spontaneous" decay of high energy particles takes place in a target free environment. When we do not know the exact nature of the decay process, we say it is "spontaneous", as our ancient ancestors did when they believed a piece of iron would go rusty "spontaneously" in the "empty" space, which was later discovered to be actually full of matter -- air.

Since the cross sections of exponential decay of high energy particles are dependent on the relative impinging velocity, it is fundamentally incorrect to ascribe such velocity dependence to relativistic time dilation.

For particles with lifetimes shorter than 1 ns, even the stopping length method, either with an emulsion or with a coincidence circuit, fails to work. The lifetime measurement is then indirectly done by measuring the widths of the energy spectrum of a particle. The lifetime is taken as the reciprocal of the line width in the energy spectrum according to the uncertainty principle, a notion that did not quite suit Einstein's taste.

6.2 Methodological

The methodologies adopted in the various lifetime measurements are by no means unquestionable. In the experiments of Rossi et al.[37,38], the lifetime of mesons was measured by counting the surviving mesons at different heights in the atmosphere. The counting rates, however, were corrected for the extensive showers and for the ionization loses. All these processes may be velocity dependent. The ionization cross section was most likely decreasing as the velocity increased due to the shorter interaction time. The velocity dependence of the cross sections of all the participating processes was then ascribed to the relativistic effect on the lifetime of the meson due to "spontaneous disintegration"[37]. In their experiments of 1940 and 1941[37,38], it was assumed that the absorption of mesons in air is the same as that in carbon with the same mass per unit area. Such an assumption is not warranted, as the interaction potential of the target atoms in the gaseous phase might be substantially different from that in the solid phase. Moreover, the cosmic rays would certainly create mesons in the air between the altitudes of Echo Lake and Denver. The newly created mesons would be added to the number of surviving ones created above the altitude of Echo Lake and make the apparent decay time much longer. These problems existed also in other similar experiments [39].

Ayres et. al [44] have measured the lifetimes of positive and negative pions at relativistic velocity, and claimed "the expected time dilation of the observed lifetime agrees with the predicted value to 0.4% and provides the most precise verification of this aspect of special relativity". In their experiment , the measured decay rate was not fit into Eq. (33), but to modified equations with four free parameters:

$$R_- = A_- \; e^{-Bx} \, [\, 1 - C \, f(x)] \qquad (37)$$

$$R_+ = A_+ \; e^{-Bx} \, [\, 1 - r \, C \, f(x)] \qquad (38)$$

where the subscripts + and - refer to π^+ and π^- respectively. The constant r was the relative changes in detection efficiencies for π^+ and π^-. r = 2.4 as deduced from pulse-height spectra. According to reference [44], the form of f(x) was not very important, and it was therefore assumed that f(x)=x. The free parameters A_-, A_+, B and C were determined by least-squares fit to be $A_- = 0.674$, $A_+ = 0.624$, $B = 5.74 \times 10^{-4}$, and $C = 1.19 \times 10^{-3}$. These values were reported without units in reference [44], and they are drastically different from their theoretical values ($A_- = 1$, $A_+ = 1$ and C = 0 if Eq. (33) was followed). It was the great freedom with these free parameters that awarded the desired relativistic time dilation. The rationale for employment of these expedient free parameters, as given in [44], was far from convincing.

It would be interesting to see what the lifetimes would be if the data were honestly fitted into Eq. (33). To do this, we replace 1-Cf(x) with exp[-Cx]. Noting that $A_- = 0.674 = e^{-0.4}$, we can rewrite Eq. (37) as:

$$R_- = \exp[-0.4 - (B+C)x] = \exp[-A' \, x_{av} - (B+C)x] \qquad (39)$$

The average distance x_{av} is about 500 cm. If we assume that the above quoted B and C values are in cm^{-1} (This seems to be the most reasonable guess. These values were reported without units in reference [44]), then A' should be $8 \times 10^{-4}\ cm^{-1}$. Overall, the spatial decay constant B' would be:

$$B' = A' + B + C = 2.56 \times 10^{-3}\ cm^{-1} = 4.5\ B$$

This would translate into a lifetime of $\tau' = \tau / 4.5 = 0.22\ \tau$. Namely, if Eq. (33) is honestly obeyed, the lifetime of π^- would be only 22 percent of what was reported in reference [42]. Similar calculation shows that the lifetime of π^+ would be merely 14 percent of what was reported. This shows that the reported time dilation was more a result of a special data reduction technique with sufficient number of expedient free parameters than direct experimental evidence.

7. Conclusion

The symmetrical 2-J and 4-J experiments designed in this article have removed all arguments based on asymmetry from the discussion of the clock paradox. The 4-J experiment demonstrates that non-unique simultaneity can lead to a paradox of occurrence.   The clock paradox and the simultaneity paradox are inherent to the Lorentz transformation, or any transformation that assumes dependency of time upon space coordinates.

The around-the-world atomic clock experiment serves as a modified version of the 2-J experiment involving acceleration and a gravitational field. The theoretical calculation within the framework of general relativity given by Hafele [34] has clearly demonstrated that the time dilation is dependent on velocity, not on acceleration, and it does not depend on the past history of the journey. The symmetrical around-the-world clocks experiment (one clock flying eastward with ground velocity v while the other westward with ground velocity $-v-2R\Omega$) offers a good example of the clock paradox. The paradoxical predictions by Hafele's formula of the time dilation of a symmetrical around-the-world atomic clock experiment is a direct challenge to the credibility of their flying clock experiments.

The experimental verification of relativistic time dilation by measuring the lifetimes of high energy particles are questionable for a number of reasons: (1) The cross section of a nuclear reaction depends in general on the impact velocity. Such velocity dependence might most likely originate from the velocity dependent interaction potential and reaction time, instead of relativistic transformation; (2) The lifetime measurements of short-lived high energy particles usually involve very complicated processes, such as ionization. More often than not, these processes are velocity dependent; (3) There are too many expedient free parameters in data processing; (4) The mesons newly produced by cosmic rays during the fly in the air have not been taken into consideration.

We have analyzed theoretically and methodologically two important types of experiments - one macroscopic and one microscopic -- that claimed to be the evidence of relativistic time dilation in order to demonstrate why these claims deserve close scrutiny. It is not the author's belief, however, that we need to follow the details of all the experimental claims to come, as the paradox is a logical contradiction rather than an experimental discrepancy, and no good experiment should support a paradoxical theory.

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Figure Captions

Figure 1. The space-time diagrams of the twin travelers. Figure 1a shows the space-time diagram of the traveling brother John in the reference frame of Jack who remains on Earth. Figure 1b shows the space-time diagram of Jack in the reference frame of John, based on the principle of relativity that all motions are relative and no motion should be considered absolutely at rest. The legitimacy of Figure 1b was challenged by the kinetic school on grounds of asymmetry. The dynamic school does allow Figure 1b and tries to resolve the clock paradox by introducing a local gravitational field.

Figure 2. The position of Jack in the reference frame of traveling John. Point O is the point of departure and reunion; Point C is the turnaround. The portion between points A and B is the cruising period. Tolman [22] introduced a local gravitational field in the region between points B and C. See the main text.

Figure 3. The gravitational field designed to realize Jack's trip.

Figure 4. The symmetrical experiment of the twin brothers, John and Jack (the 2-J experiment). The twin brothers travel in opposite directions with preprogrammed symmetrical travel itineraries. The asymmetry argument is entirely removed from the clock paradox.

Figure 5. The symmetrical experiment of the quadruplet brothers, John, Joe, Jack and Jim (the 4-J experiment). The quadruplet brothers are divided into two crews sitting in two identical long shuttles: John and Joe ride on one, with Jack and Jim on the other. John and Jack sit in the front of the shuttles, while Joe and Jim sit at the rear. Each of the four brothers is equipped with an identical clock. All the clocks are synchronized at the time of departure. The itineraries of the two shuttles are preprogrammed to be perfectly symmetric. Figure 5(a) shows the event when the two brothers in the front meet, while figure 5(b) shows the event when the two brothers at the rear meet.

Figure 6. The space and time coordinates of the four brothers at the event when John meets Jim. According to the theory of special relativity, this event is not simultaneous with the event when Jack meets Joe. The picture is drawn from the point of view of John and Joe so that Jim and Jack's shuttle looks shorter, and Jack should stay on the left side of Joe according to the Lorentz contraction. However, the Lorentz transformation gives Jack's space coordinate x in the John-

Joe frame to be greater than L, contradicting the Lorentz contraction. See the main text for detailed calculation and discussion on the relativity of simultaneity.

Figure 7. The symmetrical around-the-world clock experiment. Clock A circumnavigates eastward along the equator with the ground velocity v, while clock B circumnavigates westward along the equator with the ground velocity - $(v + R\,\Omega)$, where Ω is the angular velocity of the Earth. Their linear speeds are equal in the reference frame of a remote star. Both clocks are circumnavigating at the same altitude. The two clocks are synchronized at the time of departure. The general theory of relativity predicts that the two clocks run at the same rate. This contradicts the prediction, also by the theory of relativity, that the two clocks should register different time since one is moving with respect to the other. See the main text.

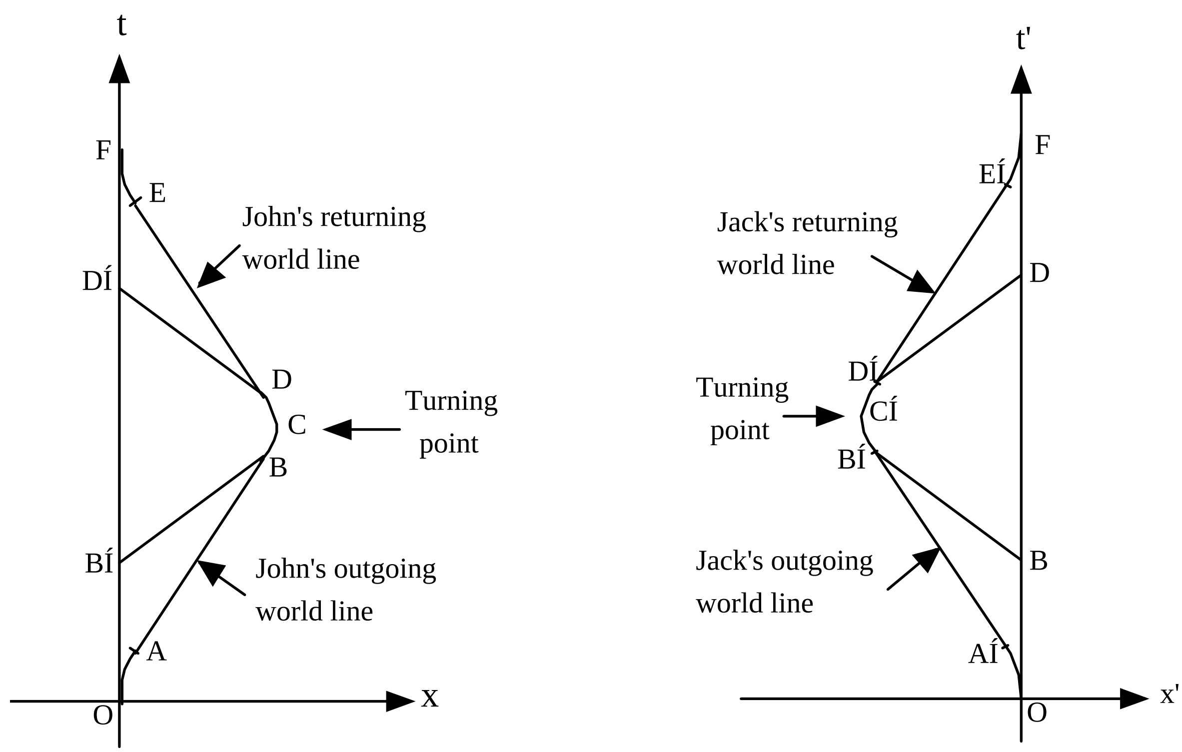
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Figure 1

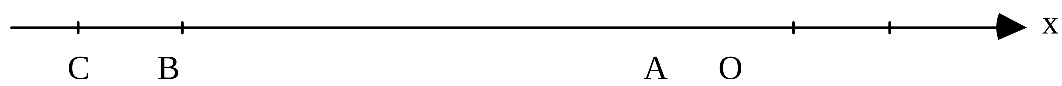


Figure 2

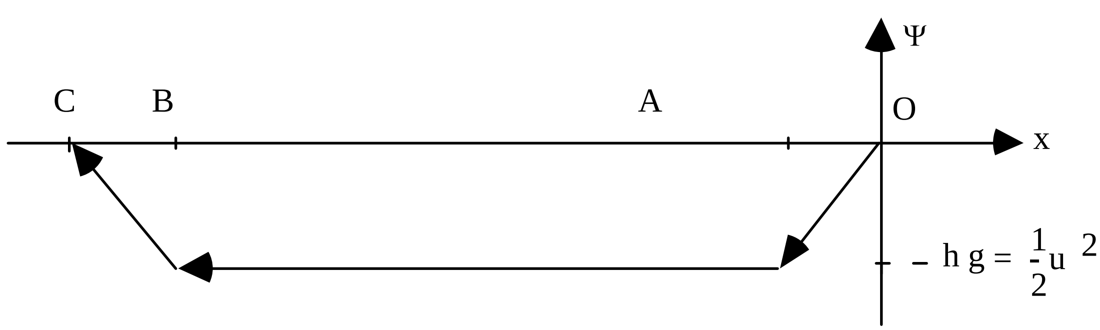


Figure 3

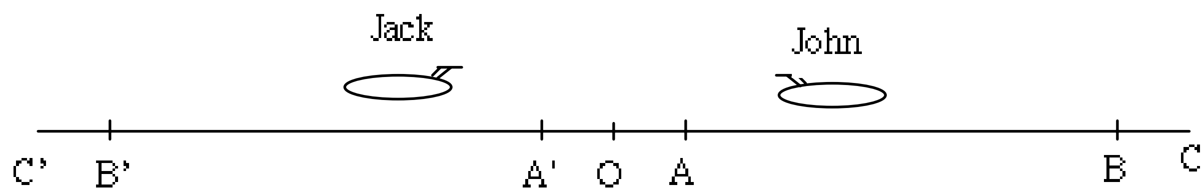
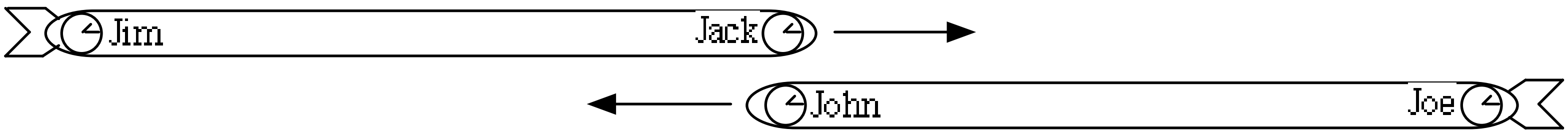


Figure 4



(a)

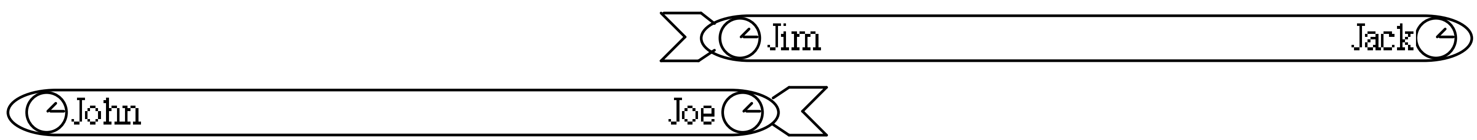


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Figure 5

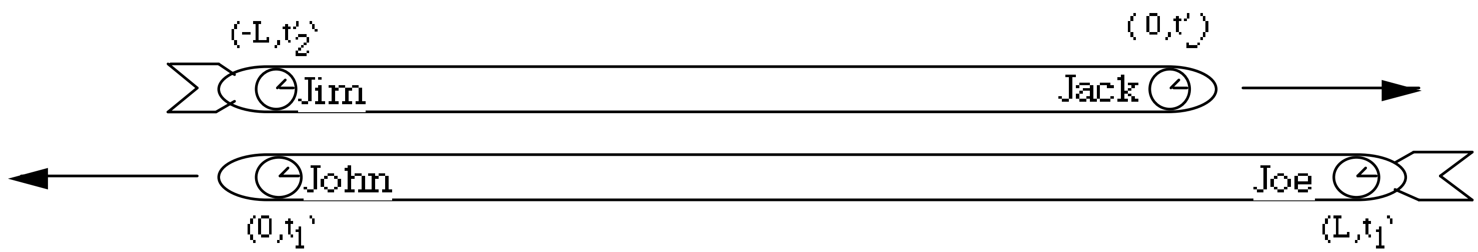

Figure 6

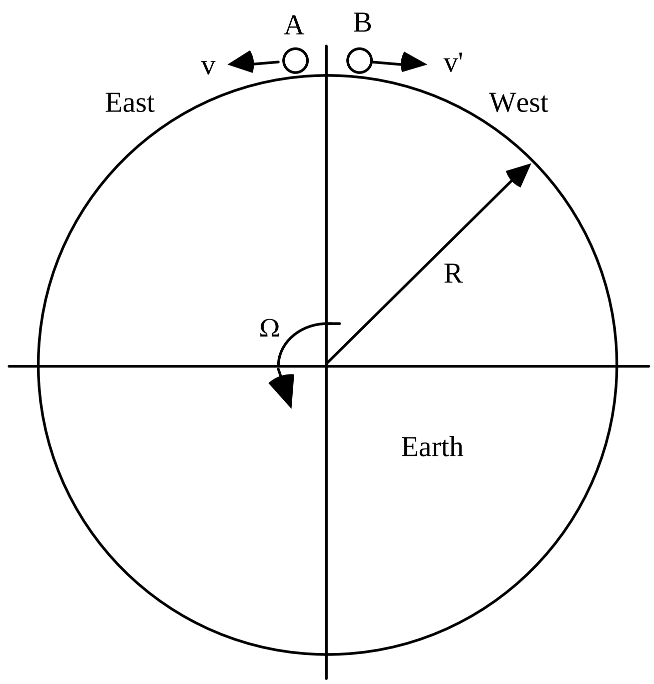

Figure 7