

EXPERIMENTAL ERROR AND DATA ANALYSIS

1. INTRODUCTION: Laboratory experiments involve taking measurements of physical quantities. No measurement of any physical quantity is ever perfectly accurate, except possibly the counting of objects. The discrepancy between the measured value and the true value of the quantity may arise from different sources. No matter how-much effort is put into refinement of technique or into improvement of the instruments, the error can only be decreased in magnitude but never eliminated entirely. The statement of the result of a measurement is not complete without an indication of how much error the measurement might contain. To obtain an experimental result with an estimate of the degree of uncertainty in the measurements, you need to know the types of errors, the ways to reduce the errors, and how to treat the data properly.

2. TYPES OF ERRORS AND HOW TO REDUCE THE ERRORS

There are three types of errors: personal, systematic, and random.

2.1 Personal Errors or mistakes arise from personal bias or carelessness in reading an instrument, in recording observations, or in calculations. Reading a value from a scale involves lining up an object with the marks on the scale. The value of the reading depends on the position of the *eye*. The reading of the object in motion depends also on the human response time.

All observations taken under the same experimental condition are equally valid and should be retained for analysis. Do not erase readings. If you must change a reading, draw a single line through it and then record the new measurement...

2.2 Systematic Errors are associated with particular instruments or techniques. Improperly calibrated instruments are one of the sources of this error. Reducing systematic errors depends on the skill of the experimenter to detect and to prevent or correct them.

2.3 Random Errors result from unknown and unpredictable variations in the experimental situation. Unpredictable fluctuations in temperature or in-line voltage are the examples of random errors. These can also be referred as accidental errors. Reduce random errors by repeating the measurement a sufficient number of times. Then taking an average or mean value, or by improving the experimental technique.

3. ACCURACY AND PRECISION

In experimental measurements there is an important distinction between accuracy and precision. The **accuracy** of a measurement signifies how close it comes to the true value, i.e., how correct it is.

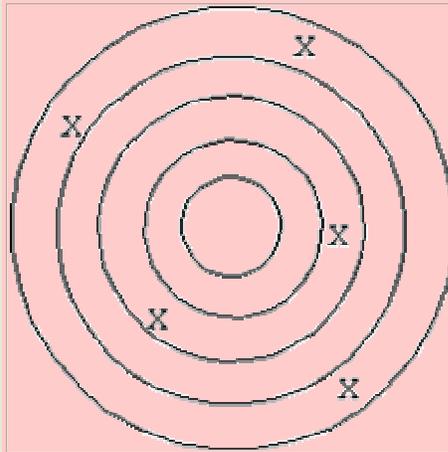
Example: If one arrow hits exactly in the center of the target, it is highly accurate.

The **precision** refers to the agreement-among--repeated-measurements, i.e., the "spread" of the measurements, or how close together they are.

Example: If six arrows hit within one centimeter of each other at one side of the target, the shooting is highly precise but not accurate.

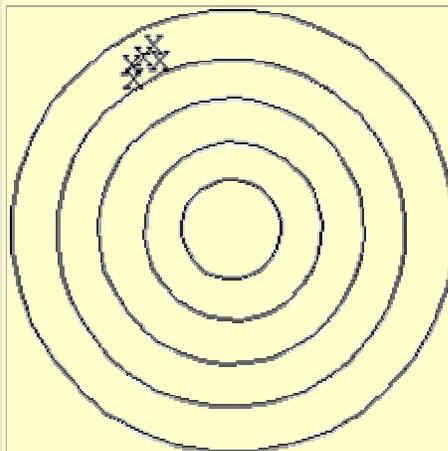
High precision does not necessarily imply high accuracy, but high precision is necessary to obtain high accuracy.

Neither Precise Nor Accurate



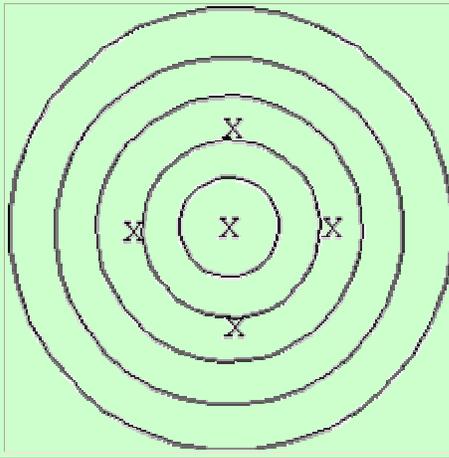
This is a random like pattern, neither precise nor accurate. The arrows are not clustered together and are not near the bull's eye.

Precise, Not Accurate



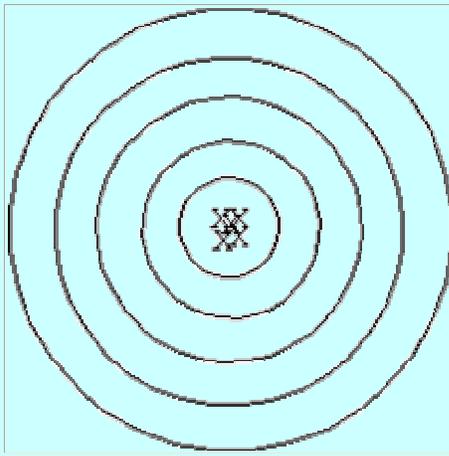
This is a precise pattern, but not accurate. The arrows are clustered together but did not hit the intended mark.

Accurate, Not Precise



This is an accurate pattern, but not precise. The arrows are not clustered, but their 'average' position is the center of the bull's eye.

Precise and Accurate



This pattern is both precise and accurate. The arrows are tightly clustered and their average position is the center of the bull's eye.

4. SIGNIFICANT FIGURES

The **significant figures** in an experimental measurement include all the numbers that can be read directly from the instrument scale plus one more estimated number. Writing the number in scientific or powers-of-10 notation shows explicitly all the significant figures in your measurement. This procedure is helpful in expressing the significant figures in very large (and very small numbers). , *Example:* The number **3564000** and **0.0001230** each have four significant figures. These are easily expressed in powers-of-10 notation as: **3.564×10^6** and **1.230×10^{-4}** .

Multiplication and Division: In the multiplication or division of two or more measurements, the number of significant figures in the final answer equals the number of the least significant figures in the measurements. *Examples:* 1. $v = dl t: 2345.2 \text{ m} / 5.2 \text{ s} = 451.0 \text{ m/s}$. The proper result is **$4.5 \times 10^2 \text{ m/s}$**
2. $d = v \times t = (20.5 \text{ m/s}) \times (30.52 \text{ s}) = 625.66 \text{ m}$. The proper result is **$6.26 \times 10^2 \text{ m}$**

Addition and Subtraction: In addition and subtraction, the number of significant figures in the final answer depends on the number of decimals in the measurements. No more decimal places than the fewest decimal places should be retained. *Examples:* (**123.56** m) + (12.351 m) = 135.911 m. (123.56 m) - (**12.3** m) = 111.26 m The results are **135.91** m and **111.3** m

5. UNCERTAINTY OR ERROR IN MEASUREMENTS

An error in a measurement is defined as the difference between the true, or accepted, value of a quantity and its measured value. To estimate the accuracy of the result, we must study the basic types of measurements.

5.1 To read the length on a ruler or the reading of any kind of pointer along a scale, the width of the scale marks and the subdivision of the scale influence the ability to read. It is easy to take a reading to the nearest scale mark with one estimated number and an error of $\pm 1/2$ of a division. *Examples:* **1.** The smallest divisions of a metric ruler are in millimeters. A position might be read as 46.2 mm. Readings of position would contain errors of ± 0.5 mm. This would be written as 46.2 ± 0.5 mm.

2. The smallest divisions of a protractor are in degrees. An angle might be measured to be 165.7° . Readings of angle would contain errors of $\pm 0.5^\circ$. This would be recorded as $(165.7 \pm 0.5)^\circ$.

5.2 Measuring a time interval is usually accomplished by starting and stopping a clock. If the clock is started and stopped by hand, most individuals are unable to time any better than ± 0.25 s, depending on the conditions and assuming that the clock is calibrated.

5.3 For electrical measurements, made with meters in which a pointer moves across a scale, the accuracy can be generally set at $\pm 3\%$ of the full scale value of the meter reading. Some indication of the accuracy of a value is given by the way it is expressed. *Examples:* 1. The full scale of an amp meter is 5 amperes. The readings of current would contain errors of ± 0.15 A. (3% of full scale) A $450.0\Omega \pm 10\%$ accuracy resistor can be expressed as $(450 \pm 45\Omega)$ _

6. RULES FOR COMBINATION OF ERRORS Let $C = C \pm \Delta C$, $A = A \pm \Delta A$ and $B = B \pm \Delta B$

Δ means error in or uncertainty in a number. NOT CHANGE.

6.1 Addition:

$$C = A + B$$

$$C \pm \Delta C = (A \pm \Delta A) + (B \pm \Delta B) = (A + B) \pm (\Delta A + \Delta B)$$

$$C = A + B \text{ and } \pm \Delta C = \pm(\Delta A + \Delta B) \text{ or } \Delta C = \Delta A + \Delta B$$

6.2 Subtraction :

$$C = A - B$$

$$C \pm \Delta C = (A \pm \Delta A) - (B \pm \Delta B) = (A - B) \pm (\Delta A + \Delta B)$$

$$C = A - B \text{ and } \pm \Delta C = \pm(\Delta A + \Delta B) \text{ or } \Delta C = \Delta A + \Delta B$$

6.3 Multiplication:

$$C = (A) * (B)$$

$$C \pm \Delta C = (A \pm \Delta A) * (B \pm \Delta B) = A * B \pm B * \Delta A \pm A * \Delta B \pm \Delta A * \Delta B$$

$$C = A * B \text{ and } \pm \Delta C = \pm (B * \Delta A \pm A * \Delta B \pm \Delta A * \Delta B) \text{ or } \pm \Delta C = \pm (A * B) (\Delta A / A + \Delta B / B + \Delta A * \Delta B / A * B)$$

If $\Delta A / A$ and $\Delta B / B$ are much less than 1 then $(\Delta A * \Delta B) / (A * B)$ can be ignored. Then

$$C = A * B \text{ and } \Delta C = (A * B) (\Delta A / A + \Delta B / B)$$

6.4 Division :

$$C = A / B$$

$$C \pm \Delta C = (A \pm \Delta A) / (B \pm \Delta B)$$

Similarly (although not so easily) it can be shown that $C = A / B$ and

$$\Delta C / C = \Delta A / A + \Delta B / B \text{ for small errors.}$$

$$C = A / B \text{ and } \Delta C = (A / B) * (\Delta A / A + \Delta B / B)$$

7. ESTIMATION OF THE MEAN VALUE AND STANDARD DEVIATION FOR A SMALL SAMPLE.

For any number of measurements n , an estimate of mean value can be calculated from the following equation:

$$Mean = \sum_{i=1}^n X_i / n$$

where Σ means sum of all the numbers.

For a small number of measurements n , not more than 10, an estimate of the standard deviation (the spread of the number about the mean) can be based simply on the range, which can be found as follows.

Range = (largest reading) - (smallest reading)

Standard deviation = Range / \sqrt{n}

Examples:

Three distances were measured to be 6.25 m, 6.31 m and 6.20 m.

The mean distance = $(6.25 + 6.31 + 6.20) / 3 = 18.76 / 3 = 6.2533$ m -

Standard deviation = $(6.31\text{m} - 6.20\text{m}) / \sqrt{3} = 0.11\text{m} / \sqrt{3} = 0.0635$

m. The statement of the distance is (6.25 ± 0.06) m.

8. GRAPHS

Graphs provide a visual representation of data and provide a faster form of communication than tables of data. Graphs may also be used as an analytical tool to determine the value of some quantity. The following suggestions should help you produce meaningful graphs.

a. A graph should have a self-explanatory title and both axes should be clearly labeled including the units of the quantities.

b. The independent variable is plotted on the horizontal or x-axis and the dependent variable on the vertical or y-axis.

The independent variable is the one changed by the experimenter to determine the effects on the dependent variable.

c. The scale should be chosen so that the data occupies as much of the available space as possible. The scale on each axis should be divided evenly. But the scales on both axes do not have to be the same size and in some cases do not have to start at zero.

d. Each point on a graph should have the possible error indicated. If only one variable has error this may be done as an error bar. If both have errors then two bars or a rectangular with the data point at the center may be used.

Different shapes, such as circles or triangles with a data point at each center, may be used to distinguish between different groups of data on the same graph.

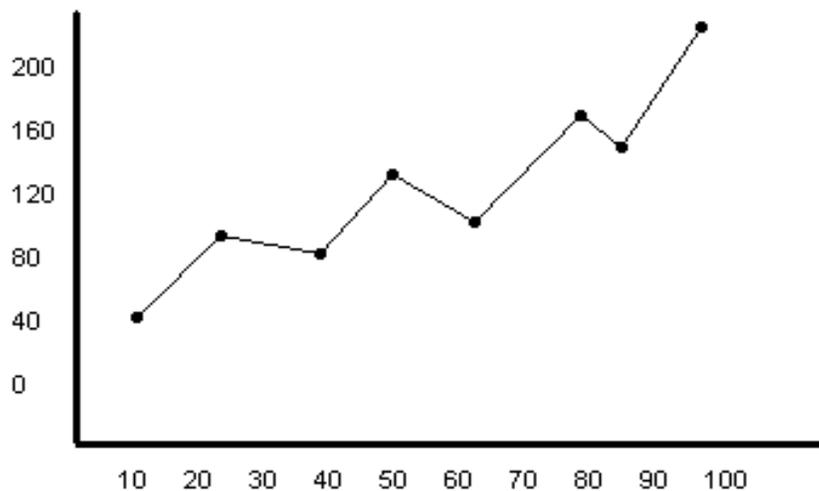
e. Do not connect the points with straight line segments. If the points seem to lie along a straight line then try to draw a single straight line which passes as close to as many points as possible.

f. If the data does not represent a straight line, it may be informative to plot functions of the variables on semi-log or log-log scale in order to obtain a straight line graph.

References:

1. EVANS, Robley, D. : The Atomic Nucleus
McGraw-Hill Book Company, Inc. 1955
2. DAVIS, Joel L. and PETERSON, Randolph:
Treatment of Data & Errors in the Physics Laboratory

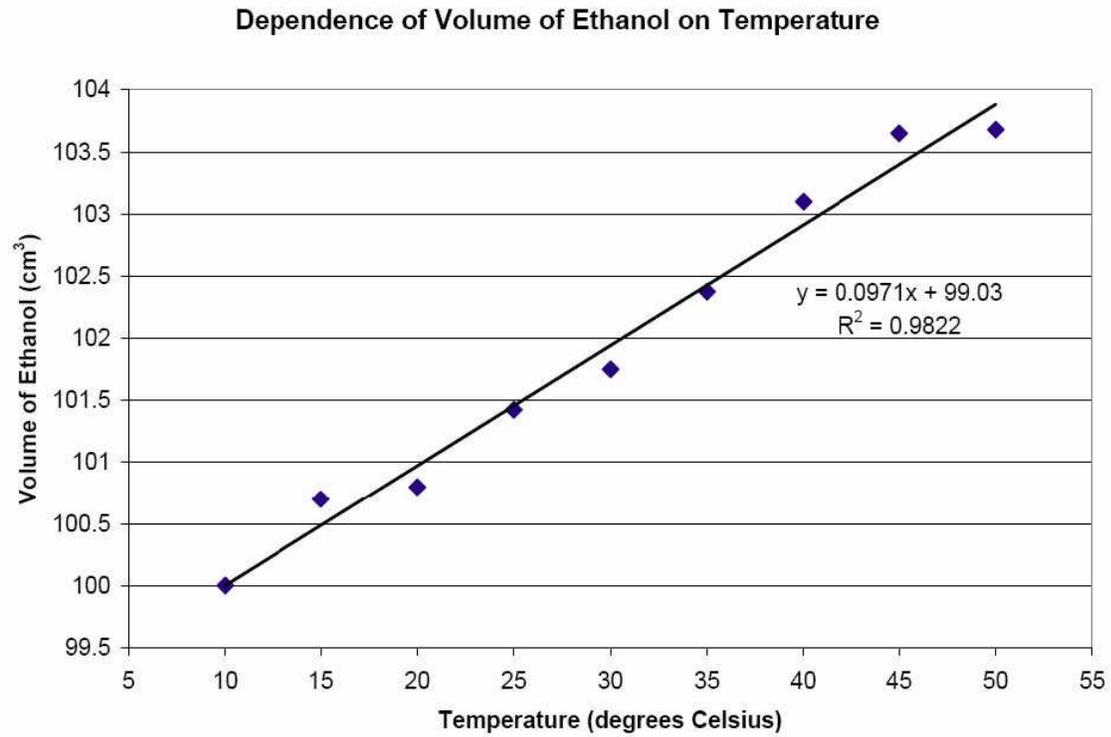
An Example of a Bad Graph



Let's see what's wrong with this graph:

- There's no title. What's it a graph of? Who knows?
- There are no labels on the x or y axis. What are those numbers? Who knows?
- There are no units on the x or y axis or in the Title. Is this a graph of speed in miles per hour or a graph of temperature in Kelvins? Who can tell?
- Somebody played "connect the dots". This should be a nice straight line which goes through the points or a curve that tends to follow them

An Example of a Good Graph



Doesn't the clarity and beauty of this graph just make you want to cry? Well, maybe that's overstating it a little bit, but it sure does make more sense than the first one, doesn't it? I'm starting to mist up right now.