<u>**Measurement and Error Lab**</u>

Object: To learn a simple method of how error propagates in the calculation of all quantities based on physical measurement.

Theory: In many types of measurement processes, the uncertainties (resulting from the limitations of the equipment) are randomly distributed. That is, they are "bunched" about some average value according to a Gaussian (or "normal") distribution. In this case, it can be shown 1 that if f is a quantity depending upon measurements x, y, z, etc., then the error* in each of the measurements will produce an error in f given by f in the formula below:

$$(\Delta f)^2 = [\delta f/\delta x]^2 (\Delta x)^2 \; + \; [\delta f/\delta y]^2 (\Delta y)^2 + \; [\delta f/\delta z]^2 (\Delta z)^2 + \; \ldots\ldots$$

where the variable $\mathbf{\Delta x}$ represents the <u>uncertainty in the measured value </u>of x, etc. The derivatives are most often evaluated using the average values of the independent variables. This relation is known as the **Gaussian error formula.** This formula is put to its best use when more than one independent measurement can be made for the variables, and the standard deviation of x. for instance, is used for $\mathbf{\Delta x}$, and similarly for the other variables on which **f** depends.

When only a single measurement can be made of the independent variables, then often a simpler approach for calculating error propagation is useful:

- **for quantities which are to be added or subtracted, the absolute errors in those quantities always add;**

- **for quantities which are to be multiplied or divided, the relative errors in those quantities always add.**

Some examples will help clarify these rules.

(A) Suppose we wish to add together two lengths, 2.30 $\pm$ 0.08 m and 0.60$\pm$ 0.05 m. The result is 2.90 $\pm$ 0.13 m. If we had subtracted instead of added, the error would still be $\pm$ 0.13 m. This allows for the "worst case scenario," which is demanded by scientific integrity.

(B) If we wish to multiply the lengths (e.g., to calculate an area A) then we must add the relative errors to find the relative error in the result

$$\Delta A/A = 0.08/2.30 \quad + 0.05/0.60 \approx 0.118$$

from which we find

$$\Delta A = 0.118A \quad = 0.118(1.38) + 0.1628 \approx 0.16 \text{ m}^2$$

(Note that we have rounded off our answer in the final step: since the raw data is uncertain in the hundredths place, it makes no sense to say that the product of these values is uncertain to the thousandths place! We state the result as A = 1.38 $\pm$ 0.16 m^2. Quotients are found in the same way.)
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* Please keep in mind that the use of the word "error" is simply a shorthand way of writing "uncertainty ." Mistakes due to incorrect calculations, sloppiness in using a measurement tool, or other types of carelessness are <u>mistakes,</u> not errors, and have no place in the careful work expected of you in this laboratory!

This method of dealing with experimental uncertainty is based on the simple differential learned in first-year calculus: for f = f(x, y, z, ...),

$$df(x,y,z,.......) = [\delta f/\delta x](dx) + [\delta f/\delta y](d\ y) + [\delta f/\delta z](d\ z) + ...$$

Assuming dx, dy, and dz… are very small compared to x, y, x… They can be written as Δx, Δy, Δz… etc. the uncertains in the measurements

The instructor may give other examples.

Procedure: Each group will be given two objects (geometrical solids) for which they must measure --as accurately as possible with the given measurement apparatus -the surface area and the volume, and the corresponding uncertainties in these quantities.
a) Each person in a group should make independent measurements of the relevant variables, and record his/her values and the associated uncertainties alone.
   Do Not Give Your Numerical <u>Values to Any Other Person</u> !!!
b) Use the formula below (get the ones you need which are missing!) to find the total surface area and volume for your figures.
c) Calculate the errors in the surface area and the volume. **You must show all work here. d) State your answers and the errors to the appropriate <u>number of significant digits.</u>**

Useful formulae:

~	SURFACE AREA	VOLUME
Solid Sphere	$4\pi R^2$	$(4/3)\pi R^3$
Hemisphere		
Cylinder		
Prism		
Pyramid (3 slanted sides)		(1/3) Bh
Prismatiod (Like the Egyptian pyramids) 4 slanted sides		(1/6)h(B+M)
Cone	$\pi R(R+s)$	$(1/3)R^2 h$
Hexagonal prism		

Note. B= area of the base, h = altitude, s = slant height. M = area of the mid section.
Surface areas include all surfaces of the solid ie both the top and bottom surfaces of the cylinder for example, not just the vertical area. Mensuration formulas for other solids may be found in the **<u>Handbook of Chemistry and Physics</u>** as well as other math reference books.