

## STANDING WAVES

**Objective:** To verify the relationship between wave velocity, wavelength, and frequency of a transverse wave.

**Apparatus:** Magnetic oscillator, string, mass hanger and assorted masses, pulley, a 2-meter stick, a balance, and a large C clamp.

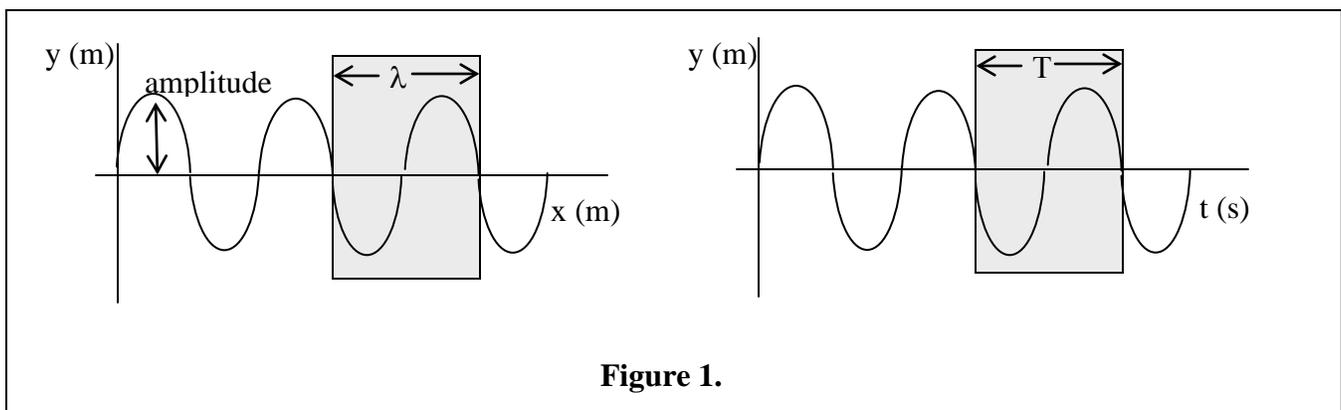
**Theory:**

A wave is a traveling disturbance that carries energy from place to place, and can be classified as one of two basic types, transverse or longitudinal. If you are holding one end of a string and move it up and down in simple harmonic motion, you will generate a transverse wave that moves down the length of the string; thus, for a transverse wave, the direction of particle vibration is perpendicular to the direction that the wave propagates in.

A transverse wave can be described by the position of the particles in the wave at a given time, and also by how the position of the particles in the wave varies in time. As shown in figure 1, the amplitude of a wave is the maximum vertical displacement of a particle in the string from its equilibrium position. The wavelength,  $\lambda$ , is the horizontal length of one cycle of the wave (between two successive crests, troughs, or equivalent points) and has standard units of distance, m. The wave period,  $T$ , is defined as the amount of time for one complete cycle of vibration for each particle in the wave (in standard units of seconds), and is also the time required for the wave to travel one wavelength. The frequency of a wave is the reciprocal of the period, and as such, has standard units of  $1/s$ , defined as a hertz (Hz). The frequency of a wave is the number of complete cycles per second that it undergoes. For any periodic wave, the wave velocity,  $v$ , is given by

$$v (m/s) = \frac{\lambda (m)}{T (s)} \quad \text{or} \quad v = f\lambda \quad \text{(eq. 1)}$$

where  $f$  is the frequency in hertz and  $\lambda$  is the wavelength of the wave in meters.



The properties of the string through which a wave travels will also determine the speed of the wave. Newton's second law of motion tells us that a stronger force produces a greater acceleration, and thus a faster moving wave. Thus, the ability of a particle on the string to pull on the particle next to it

will be directly proportional to the tension in the string, because the greater the tension in the string, the greater the pulling force from one particle to the next one. In addition, the mass of the adjacent particle on the string will affect how quickly it responds to the pull from the previous particle. Thus the speed of a wave on a string will be inversely proportional to its linear density,  $\mu$ , which is the mass per unit length of the string ( $\mu = m/L$ ). In fact, the velocity of the wave is proportional to the square root of these quantities:

$$v = \sqrt{\frac{F}{\mu}} \quad (\text{eq. 2})$$

where  $F$  is the tension in the string.

Rearranging equation 1 to solve for  $v$  and combining this with equation 2 yields  $f\lambda = \sqrt{\frac{F}{\mu}}$ .

Solving for the force, we find:

$$F = \mu f^2 \lambda^2 \quad (\text{eq. 3})$$

Data on a plot of  $F$  vs.  $\lambda^2$  could theoretically be fit by a linear function of the form  $y = mx + b$ , where the slope of the best fit line would be  $\mu f^2$ . We can therefore determine the frequency of the oscillation that produced the wave by:

$$f = \sqrt{\frac{\text{slope}}{\mu}} \quad (\text{eq. 4})$$

Thus, the frequency of oscillation can be determined if we can measure the force, or tension in the string, as a function of different wavelengths. The measurement of force and wavelength are attainable if we make use of the conditions for resonance and producing a standing wave on a string.

If a stretched string is clamped at both ends, when a wave traveling in one direction reaches the fixed end, it will reflect and travel in the opposite direction. The incident and the reflected waves will combine according to the principle of linear superposition. If the proper amount of tension is introduced into the string, a standing wave (or so-called stationary wave) can be produced, as shown in Figure 2. The standing wave is “standing” because this waveform does not appear to travel in one direction or the other, as do the individual waves that produce it. Each time the traveling wave reflects from a fixed end, it adds more energy to the string and reinforces the wave pattern. Repeated reinforcement leads to resonance, a condition of maximum energy transfer in an oscillating system, and causes a large amplitude to develop in the waveform, thus allowing us to see the wave and measure its wavelength. As shown in figure 2, we can produce a standing wave pattern by adjusting the tension in the string appropriately. We increase the tension in the string by applying a force,  $F$ , which can be the weight (mg) of a mass hanging vertically from a pulley at the fixed end of the string.

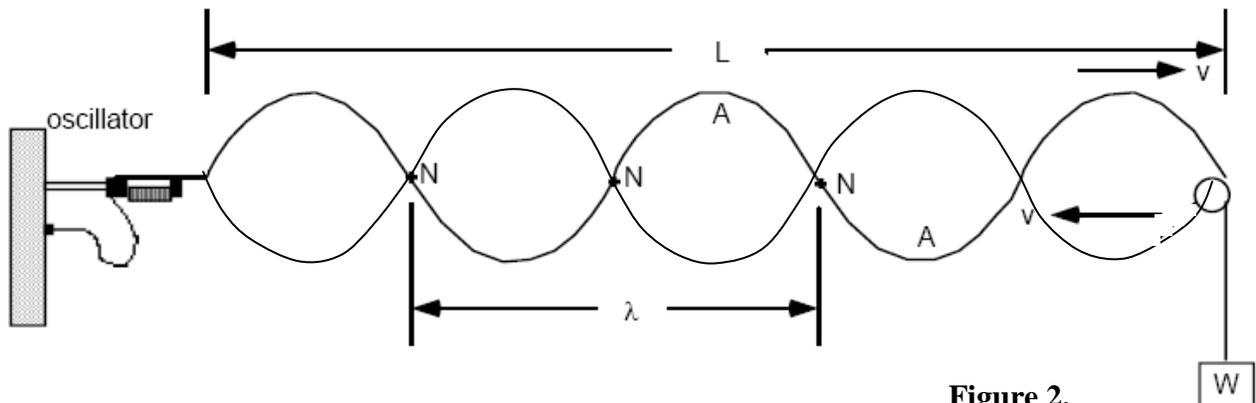


Figure 2.

Along the standing wave shown in figure 2 there are regions of no vibration called nodes (N) and regions of maximum vibration called antinodes (A). The segments between nodes are called loops (for example, there are 5 loops shown in figure 2). The distance between two successive nodes (or antinodes) is equal to one-half of a wavelength. Since the two fixed ends of the string are nodes, then the length  $L$  of the string must contain an integer number ( $n$ ) of half-wavelengths,  $L = n (\frac{1}{2} \lambda)$ , or

$$\lambda = \frac{2L}{n} \quad (\text{eq. 5})$$

where  $n$  is the number of loops.

## Procedure and Data Analysis:

### *PART I. Determination of the wave velocities.*

1. Construct the apparatus as shown in figure 2.
  - a. Clamp the PASCO vibrator to one end of the lab table as shown below. Figure 3A
  - b. Clamp the pulley to the other end of the lab table about 1.5-2 m away, and stretch the string connected to the oscillator over it, as shown in figure 3B.
  - d. Hang the mass hanger from the loop in the end of the string as shown in figure 3C.

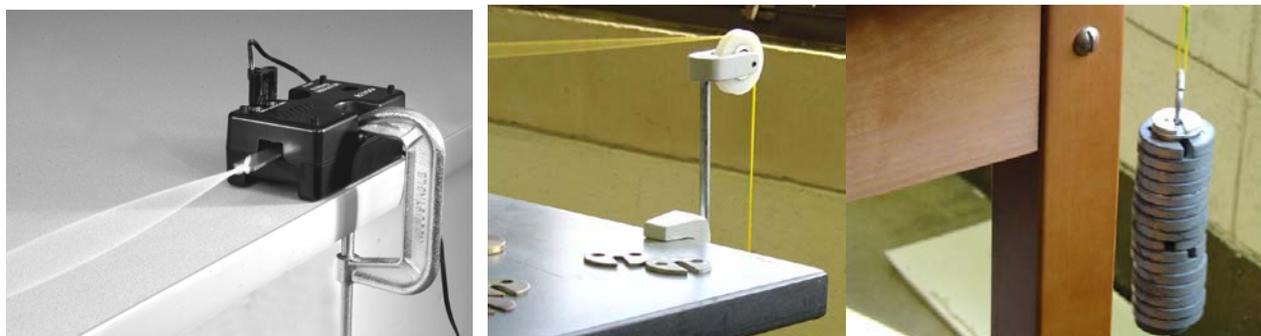


Figure 3A, 3B, 3C. Lab Setup

2. Measure the distance,  $L$ , between the end of the oscillator and the center (axel) of the pulley where the string contacts it. This will be the distance between the outermost nodes (at the fixed ends) of the standing waves. Record this on your data sheet.
3. Adjust the string's tension by adding masses until a standing wave is produced with at least 5 loops (the greater the weight, the fewer the loops.) It is easiest to start with a small amount of mass and gradually add more mass. Standing waves are formed for a specific tension (provided by the weight) only. Not just any mass will form the standing waves so you must find the exact amount.

- Hint: You may wish to estimate the correct mass to produce 5 loops to use as a starting point and gradually add or subtract weight. Combining equations 3 and 5 and solving for  $m$  yields:

$$m = \frac{4\mu f^2 L^2}{gn^2}$$

Where  $\mu$  is the linear density of the string and  $f$  is the frequency of the oscillator (ask your instructor for these values);  $L$  is the length of the string,  $g$  is the acceleration due to gravity,  $9.80 \text{ m/s}^2$ ; and  $n$  is the number of loops.

- Note: when adding mass, remember to include the mass of the hanger.

- Hint: If your calculated mass does not produce a standing wave pattern, gently pull up or down on the string to determine whether to add or subtract mass from the hanger.
- The standing wave should appear similar to the one shown in figure 4.

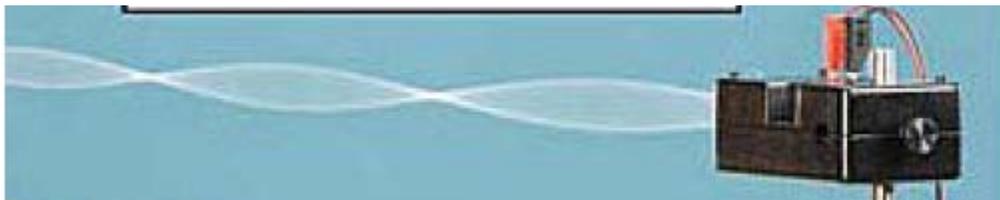


Figure 4.

4. Record the number of loops,  $n$ , in the first column of your data sheet.
5. Weigh the mass hanger and masses together on the balance and record it as the mass,  $m$ , in the second row on your data sheet.
6. Calculate the tension in the string,  $F$ , produced by the weight of the mass ( $F=mg$ ) and record this in row 3 of the data table.
7. Calculate the wavelength,  $\lambda$ , of the wave using equation 5. Record it on your data sheet.
8. Calculate the value for  $\lambda^2$  and record it.
9. Calculate the wave velocity using both equations 1 and 2 and record the values on your data sheet. Use the values of  $\mu$  and  $f$  given to you by the instructor.
10. Find the % difference between the velocities obtained from the two equations. Record this on your data table.
11. Increase the tension by adding more masses until another standing wave with fewer loops occurs.
12. Repeat steps 4-10 for two more standing waves.

### ***PART II. Experimental determination of the frequency of oscillation.***

1. Using your data for the tension on the string and the square of the wavelength for the three different standing waves, make a graph of  $F$  as a function of  $\lambda^2$ .
  - Note: You can have 4 data points for this graph: a data point from each of the 3 standing waves, and an additional data point at (0,0).
  - Note: If you are constructing the graph using Excel, please make sure that the x-data values are in column A and the y-data values are in column B. Otherwise, excel will plot the wrong coordinates on the wrong axes (the y values will be on the x-axis and vice versa).
2. Fit the data with a linear function of the form  $y=mx+b$  and get an estimate of the goodness of fit.
3. Determine the frequency,  $f$ , of the oscillator from the slope of the best-fit line, as indicated by equation 4.
4. Find the percent error between your experimentally determined frequency of the wave and the nominal value for the frequency of the oscillator.

### **Lab Report Format:**

Your lab report for this experiment should contain:

1. Pre-lab (objective, theory, sketch of the experimental set up, and procedure).

2. Neatly written copy of your experimental data sheet.
3. Sample calculations: For Part I of the experiment, show a sample calculation for (1) the tension in the string,  $F$ ; (2) the wavelength of the standing wave,  $\lambda$ ; (3 and 4) the wave velocities found by the two different equations, (5) the percent difference calculation for the difference between the two velocities. For part II of the experiment, show (6) the calculation for the frequency of the oscillator from the slope of the graph; and (7) the relative percent error from the theoretical value for the frequency of the oscillator.
4. Graphs: Graph of  $F$  vs.  $\lambda^2$ . Make sure it has a title, appropriately-labeled axes with units, and a best-fit line to the data with an equation and an estimate of the goodness of fit ( $R^2$  or MSE).
5. Results: Report your results (in complete sentences) for the values of the velocity of the waves determined by both equations for each wave in part one of the experiment. How well do they agree with each other (report the percent differences between them). Report the equation of the best fit line to your data and the goodness of fit from part II of the experiment. What was the experimentally determined value of the frequency of the wave? How well does this value agree with the nominal value of the frequency of the oscillator? (report the percent difference). Make sure all values are properly rounded and have the correct number of significant digits.
6. In your conclusions, address the answers to the discussion questions below:
  1. Comment on the wave velocity as a function of the number of loops on the wave. Does your experiment show that the two are directly proportional or inversely proportional? Explain why this should also be true theoretically (i.e., derive and report a mathematical relationship between  $v$  and  $n$ . Hint: You might start with the relationships defined in equations 1 and 5).
  2. Analyze the data points and the best fit line on your graph. What does it indicate about whether there might have been systematic or random error in the experiment? (Note: you might want to refer to the definitions of systematic and random uncertainties on the fundamentals of error analysis handout).
  3. Part II of the experiment allowed you to experimentally determine the frequency, but the value you used for the frequency in order to calculate the velocity of the wave in part I of the experiment used the nominal value. In light of this and any percent difference between the two values for the frequency, discuss which measure of the velocity in part I of the experiment might/might not have been the more accurate determination. If you find that there was not a large percent difference between the frequencies, which quantity is providing any percent difference between the two determinations of velocity?