

RESOLUTION OF FORCES

Objective: To verify that the sum of forces on an object at rest in equilibrium is zero.

Apparatus: Force table, a set of weights, weight hangers, level, and cords.

Theory:

Newton's second law of motion states that the acceleration produced by a net force on an object is directly proportional to the net force, is in the same direction as the net force, and is inversely proportional to the mass of the object:

$$\mathbf{a} = \frac{\Sigma \mathbf{F}}{m} \quad \text{or} \quad \Sigma \mathbf{F} = m \mathbf{a} \quad (\text{eq. 1})$$

where the bold faced letters represent vector quantities. For the case when an object is not undergoing an acceleration (the velocity of the object is either 0, or the object is moving at a constant velocity), then $\Sigma \mathbf{F} = 0$, which is the first condition for equilibrium. The sum of the forces, $\Sigma \mathbf{F}$, is the algebraic sum of all the individual forces:

$$\Sigma \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \dots = 0 \quad (\text{eq. 2})$$

Take for example the case of a box that is hanging motionless from a rope. The force that is due to the weight of the object, \mathbf{F}_1 , must be equal and opposite to the direction of another force, \mathbf{F}_2 , if the box is not accelerating. \mathbf{F}_2 must be the "support force", in this case, the tension in the rope that acts in the upward direction to balance the force downward. Weight is a force, and by eq. 1, the force \mathbf{F}_1 on the box downward is:

$$\mathbf{W} = m \mathbf{g} \quad (\text{eq. 3})$$

where W is the weight and g is the acceleration due to gravity, -9.8 m/s^2 , where the negative sign indicates that gravity is acting in a downward direction. By the condition for equilibrium, $\mathbf{F}_1 + \mathbf{F}_2 = 0$, or $\mathbf{W} + \mathbf{F}_2 = 0$, or $\mathbf{F}_2 = -\mathbf{W}$.

Equation 2 deals with vector quantities; breaking this down into components yields three equivalent scalar equations, each representing one of the spatial directions, x, y, or z:

$$\Sigma F_x = F_{1x} + F_{2x} + F_{3x} + \dots = 0 \quad (\text{eq. 4})$$

$$\Sigma F_y = F_{1y} + F_{2y} + F_{3y} + \dots = 0 \quad (\text{eq. 5})$$

$$\Sigma F_z = F_{1z} + F_{2z} + F_{3z} + \dots = 0 \quad (\text{eq. 6})$$

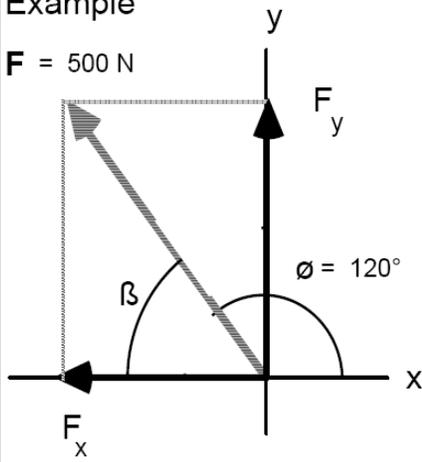
Therefore, if the magnitude and direction of a force are known, then its components in each of these directions may be found using trigonometry. The x and y components can be found by the following equations assuming that the angle θ is defined as counterclockwise from the positive x axis:

$$F_x = F \cos \theta \quad \text{and} \quad F_y = F \sin \theta \quad (\text{eq. 7 and 8})$$

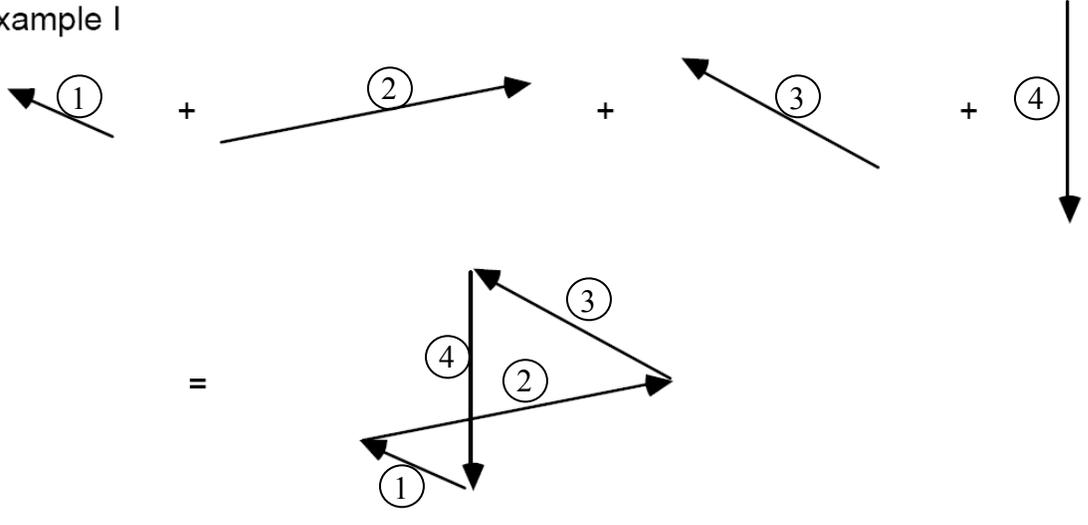
The x and y components of a vector can be summed by means of the Pythagorean theorem to produce the magnitude of the resultant vector; the angle indicating the direction of the resultant can be found by taking the inverse tangent of the ratio of the magnitude of the y and x components:

$$F = \sqrt{F_x^2 + F_y^2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) \quad \text{(eq. 9 and 10)}$$

The following example illustrates this for a two dimensional coordinate system.

<p>Example</p>  <p>$F = 500 \text{ N}$</p> <p>$\theta = 120^\circ$</p> <p>F_x</p> <p>F_y</p> <p>β</p>	$F_x = F(\cos \theta)$ $= (500 \text{ N})(\cos 120^\circ)$ $= -250 \text{ N}$ $F_y = F(\sin \theta)$ $= (500 \text{ N})(\sin 120^\circ)$ $= 433 \text{ N}$	<p style="text-align: center;">OR</p> $\beta = 180^\circ - \theta = 60^\circ$ $F_x = -F(\cos \beta)$ $= -(500 \text{ N})(\cos 60^\circ)$ $= -250 \text{ N}$ $F_y = F(\sin \beta)$ $= (500 \text{ N})(\sin 60^\circ)$ $= 433 \text{ N}$
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For equilibrium of an object that is at the origin and being pulled by the force such as shown in the example above, another vector force whose x and y components are $+F_x$ and $+F_y$, respectively, must be applied. Furthermore, if the system is in equilibrium, the force vectors will form a closed figure when they are connected, head to tail, as shown in the following example.

<p>Example I</p>	
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Procedure and Data Analysis:

There are two parts in performing this experiment. The first is to calculate the third force (its magnitude and direction) that is required for a system of three forces in equilibrium. In the second part of the experiment, we will verify that the system of all three forces is in fact in equilibrium by the use of the force table.

PART I. Calculation of a third force that produces an equilibrium condition:

1. Record the given values for the masses of objects 1 and 2, m_1 and m_2 , (in kilograms) and the associated angles, θ_1 and θ_2 , with respect to the origin.
2. Calculate the magnitude of the forces, F_1 and F_2 , that are associated with the weight of masses 1 and 2 respectively, by using equation 3. If the units of mass are kilograms and the units of g are m/s^2 , then the units for the weights should be in Newtons.
3. Calculate the x and y components of F_1 and F_2 using equations 7 and 8.
4. Use equations 4 and 5 to find the components of the third force, F_{3x} and F_{3y}
5. Use equations 9 and 10 to find the magnitude and direction (θ_3) of F_3 .
6. Calculate the mass, m_3 , needed to produce the force, F_3 , using equation 3. This is the mass that must be hung at an angle θ_3 so that an object subjected to the three forces is at equilibrium.

PART II. Verification that the system of all three forces is in equilibrium.

1. Adjust the force table to so that it is level horizontally. Make sure that the axis of rotation for every pulley is perpendicular to the radius of the force table.
2. Use the centering pin to anchor the ring to the center of the table.
3. Place pulleys at the given angles θ_1 and θ_2 . Hang the two given masses, m_1 and m_2 , at their respective angles.
4. Add m_3 to the system at the calculated angle θ_3 .
5. Verify that the ring is now in equilibrium by removing the centering pin. At the equilibrium position, the center of the table must coincide with the center of the ring after the centering pin is removed. NOTE: If the system is not in equilibrium (i.e., the ring is moving or not in the center of the table), keep the angle at θ_3 and adjust m_3 until equilibrium is reached.
6. Draw the force vectors, F_1 , F_2 , and F_3 , for the condition of equilibrium found in step 5 by inserting a piece of paper under the system and tracing the direction of the three cords which connect the ring to m_1 , m_2 , and m_3 .
7. Draw a vector diagram of the forces to scale:
 - a. Each force can be represented by a straight line of the proportional length to the magnitude of that force and the same direction as that force. In order to draw the force vectors to scale and the appropriate length, use a scaling factor so that $1 \text{ cm} = 0.49 \text{ N}$.
Ex) if $F_1 = 29.4 \text{ N}$, then the length of the vector should be:

$$29.4 \text{ N} \times \frac{1 \text{ cm}}{0.49 \text{ N}} = 60.0 \text{ cm}$$
 Record the values for the length of your scaled vectors, denoted respectively for F_1 , F_2 , and F_3 , as a, b, and c on your data sheet.
 - b. Draw the scaled vectors together with the appropriate angles, θ , so that they are added head to tail (the arrow end of one vector touches the straight end of the next vector, such as shown in Figure 2).
 - c. The vector diagram will form a triangle when the three forces are in equilibrium.
8. If there are experimental measurement and random errors, the vector diagram will have a gap between the starting point of the first vector and the tip of the last vector (the final point). Draw a straight line from the starting point to the final point. This is the fourth vector which represents the error vector. See figure 3 (below).
9. Use the same scale as above to find the length of the error vector in Newtons, and calculate the mass in kilograms to which this force would correspond. Record this as Δm_3 on your data sheet.

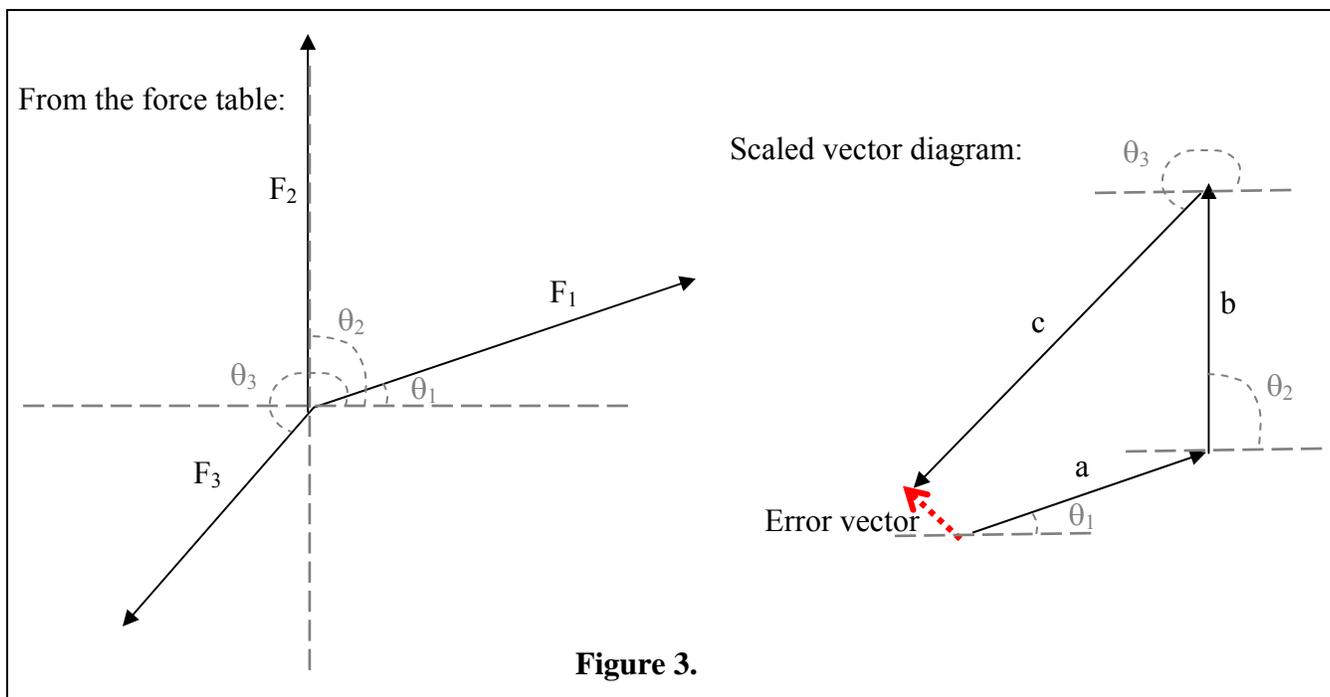


Figure 3.

Discussion Questions:

1. For our system of three forces in equilibrium, the ring on the force table was in equilibrium. In addition, each mass was in equilibrium. What were the forces that acted on m_1 ?
2. If our system had been composed of two masses instead of three, is there any angle other than $\theta=180$ degrees for m_2 with respect to m_1 possible so that the system could be in equilibrium? Explain why or why not based on trigonometry.
3. For calculations with an ideal system, we usually assume that pulley and rope systems are massless and frictionless. In the specific system for three masses with the experiment that we performed, is it possible that the mass of the cords and friction against the pulleys played any role in our determined value for m_3 and the error associated with it? Why or why not?

Lab Report Format:

Your lab report for this experiment should contain:

1. Pre-lab (objective, theory, and procedure).
2. Neatly written experimental data pages, including (a) the data sheet and (b) your scaled vector diagram made in part 2 of the experiment.
3. Sample calculations: For this lab, you need to show your entire calculations for part 1 of the experiment, an example of the scaling factor for the vector diagram, and your calculations for Δm_3 .
4. Results: Did your results meet the objective of the experiment? State your results (in complete sentences) for $m_3 \pm \Delta m_3$. Make sure all values are properly rounded and have the correct number of significant digits.
5. Conclusions: Address the equilibrium conditions in our experiment by answering the three discussion questions.