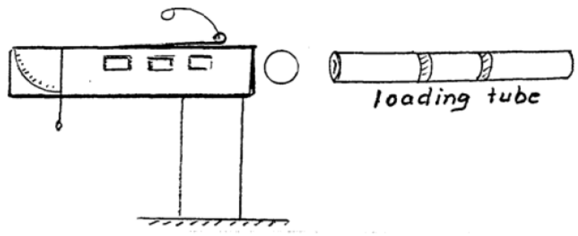
PROJECTILE MOTION

Objective: To calculate the initial velocity of a projectile and verify the equations of projectile motion.

Apparatus: Spring gun with ball, plumb bob, level, meter stick, target paper, tape and carbon paper.



Theory: Projectile motion refers to the motion of an object in a vertical plane under the influence of gravity, but is just an example of the equations of motion when applied in two dimensions. The path of a projectile is parabolic as shown in figure 1.

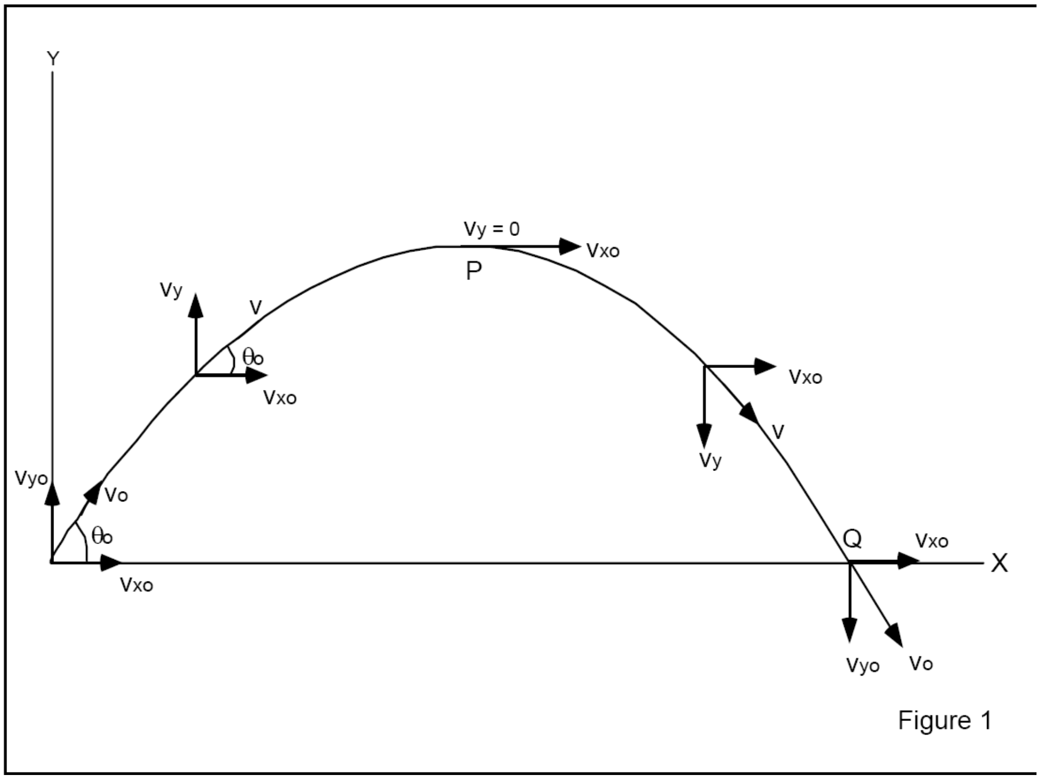


Figure 1

Initially, an object is fired from the origin, O, with initial velocity, v_0, at an angle, θ_0, with the x-axis. By trigonometry, the initial velocity can be broken down into x and y components, given by:

$$v_{0x} = v_0\cos\theta_0 \qquad \textbf{(eq. 1)}$$

and

$$v_{0y} = v_0\sin\theta_0 \qquad \textbf{(eq. 2)}$$

While the projectile is in the air, its vector velocity, **v**, changes. The speed of the object changes in time, as well as its resultant direction. If we ignore air resistance, then no force is acting on the object in the x direction, therefore there is no acceleration in this direction. If we only consider the motion of the projectile that occurs in the x direction, the kinematic equation of motion $x = v_0t + \frac{1}{2}at^2$ becomes:

$$x = v_{0x}t = (v_0\cos\theta_0)t \qquad \textbf{(eq. 3)}$$

where we note that the velocity in the x direction is always constant:

$$v_x = v_0\cos\theta = v_{0x} = \text{constant} \qquad \textbf{(eq. 4)}$$

These equations allow us to calculate the displacement of the projectile in the x direction and its x component of the velocity at any given time.

Because the force of the Earth's gravity acts on the projectile, there is an acceleration in the y direction, which is just the acceleration due to gravity, $g = 9.81\ m/s^2$. The kinematic equations of motion for the motion of the object that allow us to calculate the velocity and position of the object with respect to the y direction are:

$$v_y = v_{0y} - gt = v_0\sin\theta_0 - gt \qquad \textbf{(eq. 5)}$$

and

$$y = v_{0y}t - \frac{1}{2}gt^2 = (v_0\sin\theta_0)t - \frac{1}{2}gt^2 \qquad \textbf{(eq. 6)}$$

If the projectile is fired from a distance above the origin of the coordinate system (shown in Figure 2), the relationship between the total time the projectile is in the air, t, the distance traveled, x = OQ, and

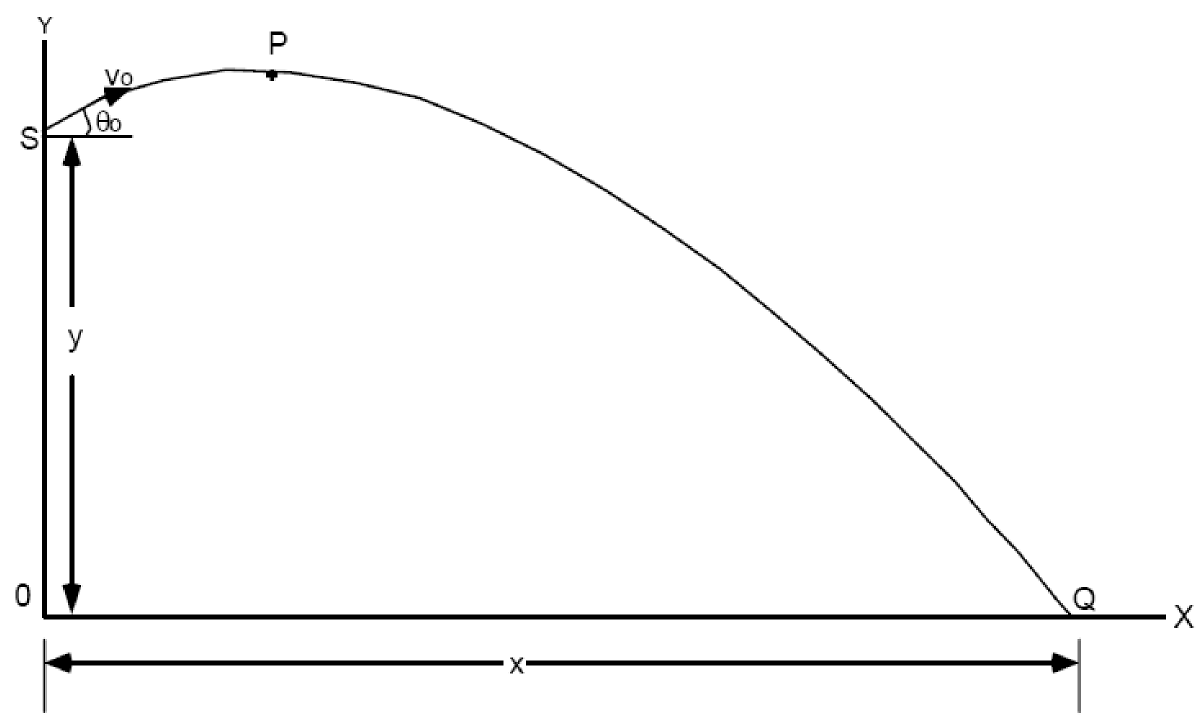


Figure 2

the y-coordinate of initial position, y = OS, can be written as follows. Rearranging equations 3 and 6 yield:

$$t = \frac{x}{v_{0x}} \qquad \textbf{(eq. 7)}$$

$$\tfrac{1}{2}gt^2 - v_{0y}t - y = 0 \qquad \textbf{(eq. 8)}$$

Equation 8 is in the form of a quadratic equation, so the solution for t would be:

$$t = \frac{v_{0y} \pm \sqrt{v_{oy}^{\ 2} + 2gy}}{g} \qquad \textbf{(eq. 9)}$$

Note that in equation 9, if the gun is fired horizontally, then there is no component of the initial velocity in the y direction; for this case, v_{0y}=0 and the equation becomes:

$$t = \sqrt{\frac{2y}{g}} \qquad \text{(if y is the absolute value of the displacement)} \qquad \textbf{(eq. 10)}$$

If the projectile is not fired horizontally, we have to discriminate between the two different solutions to this equation. Considering the path of the projectile shown in figure 1, we see that there are two times when the projectile may be at a specific y position, one on the "up" portion of the path, and one on the "down" portion of the path. For the situation shown in figure 2, a "negative time" result would correspond to the given y displacement before the projectile was fired.

In the first part of this experiment, we will be firing a projectile from the spring-loaded gun at an angle of θ_0=0 and measuring the range of the projectile (the distance OQ) in order to determine the muzzle velocity of the gun. In part 2 of the experiment, we will use this determined value of v_0 to predict the range of a projectile when the gun is fired at a different angle, θ_0=15°.

Procedure and Data Analysis:

To set the spring gun: Use a C-clamp to hold the spring gun firm to a lab table facing the wall at the far edge from the wall.

Part A: *Determining the initial speed of the ball, v_0:*

1. Set the spring gun in the horizontal position (in this case, θ_0 = 0).

2. Mark a point on the table vertically downward from the center of the ball where it will be once it is loaded. Measure the vertical distance from the marked point on the table to the horizontal line tangent to the bottom of the ball. This distance will be the height of the ball, called y_A. Include the uncertainty in the measurement, Δy_A, which will be half of the smallest increment that you can measure on your meter stick.

3. To load the gun, use the loading tube to push the ball in the barrel to the "Short range" position. At this point the gun is ready to fire.

4. Test fire the ball, noting the general area where it lands. Tape a piece of target paper in that area and lay a sheet of carbon paper over top of the target paper. Do not tape the carbon paper to the target paper, floor, or table.

5. Fire the gun ($\theta_0 = 0$) at least three times so that the ball, which is in projectile motion, lands on the carbon paper and leaves a mark on the target paper each time.

6. Measure the horizontal distance, x, from the point marked in step 2 to the landing points marked in step 5. These distances are called the ranges (x_1, x_2, x_3, ...).

7. Calculate the average range, x_A, and the standard deviation of the three points, Δx_A. Using the standard deviation as our measure of uncertainty in this case denotes the random error from uncontrolled variables in the experiment.

8. Use equation 10 to find the total time of flight for the projectile, t_A. (Use g as +9.8 m/s^2)

9. Use equation 3 to find the x component of the initial velocity of the projectile. Since the gun was fired horizontally in this case, the x component of the initial velocity is the muzzle velocity. Record this value of for v_0.

10. Calculate the experimental uncertainty in your value for the muzzle velocity, Δv_0. It can be shown by propagation of error techniques using calculus that:

$$\Delta v_0 = \left(\frac{1}{t}\right)\Delta x + \left(\frac{x}{4y}\right)\sqrt{\frac{2g}{y}}(\Delta y)$$

Part B: *Verification of equations for projectile motion through comparison of the calculated range and experimental range:*

1. Arrange the spring gun to fire at an angle θ_0 of approximately 15°. Record the actual value of θ used for your experiment.

2. Mark a point on the table vertically downward from the center of the ball where it will be once it is loaded. Measure the vertical distance from the marked point on the table to the horizontal line tangent to the bottom of the ball. This distance will be the height of the ball, called y_B.

3. Using your determined value for the muzzle velocity, v_0, and equations 1 and 2, find the x-component of the ball's initial speed, v_{0x}, and the y-component of the ball's initial speed, v_{0y}.

4. Using equation 9, calculate the ball's total time of travel from the time of firing to the time that it landed at the vertical position y, denoted as t_B.

5. Use equation 3 to determine the range, or horizontal distance, x, the projectile will travel. This is the calculated range x_{calc}.

6. Test fire the ball, noting the general area where it lands. Tape a piece of target paper in that area and lay a sheet of carbon paper over top of the target paper. Do not tape the carbon paper to the target paper, floor, or table.

7. Fire the gun three times so that the ball lands on the carbon paper and leaves a mark on the target paper each time.

8. Measure the horizontal distance, x, from the point marked in step 2 to the landing points marked in step 7. Record these ranges (x_1, x_2, x_3, ...).

9. Find the average experimental range, x, denoted as x_{exp}, and the standard deviation, Δx_{exp}, Record the values on your data sheet as $x_{exp} \pm \Delta x_{exp}$.

10. Find the percent difference between x_{cal} and x_{exp} using $\frac{x_{\exp} - x_{calc}}{x_{calc}} \times 100\%$

Discussion Questions:

1. We calculated an uncertainty in the muzzle velocity, Δv_0, that represents the range of values v_{0min} to v_{0max}, (where $v_{0min} = v_0 - \Delta v_0$ and $v_{0max} = v_0 + \Delta v_0$) in which the muzzle velocity probably falls. This uncertainty in v_0 would propagate through the calculation for the predicted range in part B of the experiment, producing a range of positions where we could be more certain that the projectile would land. Calculate the range, x_{calc}, using the value of v_0 as v_{0min} and v_{0max} to estimate a range of possible landing positions. How does the experimental range, x_{exp}, agree with the new range of values for x_{calc}?

2. In part B of the experiment, the initial firing angle, θ_0, was set at ~ 15 degrees. If the projectile had been fired at 75 degrees, what would the predicted range, x_{calc}, and time of flight, t, have been if the ball landed at an equal height to where it was fired (the total y displacement is 0)?

3. If part B of your experiment were set up like Figure 1, such that the projectile landed at the same vertical position from which it was fired, what would the velocity of the projectile have been (1) at the top of the path (labeled point P on the diagram) and (2) at the landing spot (labeled Q in figure 1). Remember, velocity is a vector, and should have both a magnitude and a direction designated with it.

Lab Report Format:

Your lab report for this experiment should contain:

1. Pre-lab (objective, theory, sketch of the experimental setup, and procedure).
2. Neatly written experimental data pages.
3. Sample calculations: For this lab, an example needs to be shown for calculation of an average and standard deviation for an experimentally found range (i.e., x_A and Δx_A): t_A, v_0, Δv_0, v_{0x} and v_{0y}, t_B, x_{calc}, and the percent difference between x_{calc} and x_{exp}.
4. Results: State your results (in a complete sentence) for the muzzle velocity and its associated uncertainty, x_{calc} and $x_{exp} \pm \Delta x_{exp}$. Make sure all values are properly rounded and have the correct

number of significant digits. Compare your results for the experimental and calculated ranges in part B of the experiment by giving the percent difference between the two. Assess what kinds of experimental variables could have caused this percent difference (that is, what physical things could have caused random variation in the data and/or a systematic error).

5. Conclusions: Address how propagation of error of the measurement of the muzzle velocity affects your calculations of the predicted range by answering discussion question 1. Use your answers to discussion questions 2 and 3 to delineate a description of motion for the projectile as it was fired in your experiment (question 3) and your predicted results for a different experiment (question 2).