Experiment 12. Electric Resonance

Objective:

To study electric resonance and learn how to obtain the band width and the quality factor.

Apparatus:

Solenoid, capacitor, resistor, oscilloscope, signal generator.

Theory:

For a circuit containing a resistor, a capacitor and an inductor connected in series, as shown in Fig.1, the current I is related to the voltage V of the AC power supply by

$$I / V = 1 / Z \quad (1)$$

with

$$Z= (R^2+(X_L^2-X_C^2))^{-1/2} \quad (2)$$

where R, X_L and X_C are the resistance, the inductive and the capacitive reactances respectively. Z is called the impedance of the circuit.

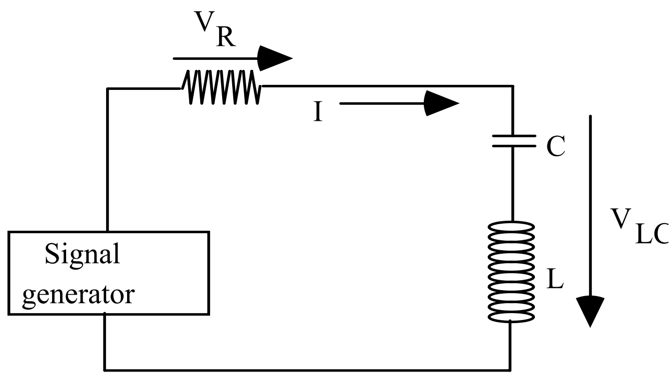

Figure 1

Since the impedance contains the reactances, it is a function of the frequency. If we plot I/V versus the frequency f, we get a Lorenzian curve depicted in Fig.(2). The curve approaches zero when the frequency approaches either infinity (when X_L is infinity) or zero (when X_C is infinity). The impedance at the two extremes is infinity which results in zero current. The current is non zero at any other finite frequency, with a peak at certain frequency f_o called the resonance frequency.

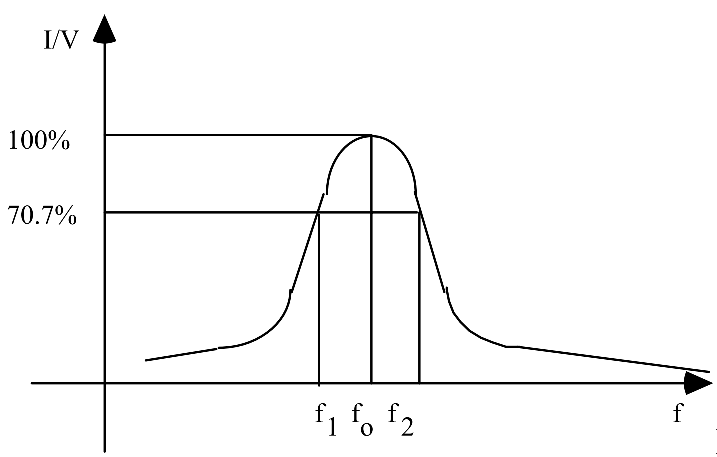

Figure 2. The resonance curve

The resonance frequency f_o can be obtained by inspecting Eq.(2). Since the resonance takes place when the impedance Z is the minimum, we let the total reactance equal to zero:

$$X_L - X_C = \omega_o L - 1 / (\omega_o C) = 0$$

$$\omega_o = \frac{1}{\sqrt{LC}} \quad (3)$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} \quad (4)$$

In telecommunication, it is also desirable to know how many different frequency channels can be placed within certain band without interference, i.e., how "wide" are the resonance peaks. Since the resonance peak covers theoretically the whole frequency spectrum and no clear cut board lines can be defined, the width of the resonance peak is defined to be the difference between two frequencies f_1 and f_2 when the amplitude of the current (or voltage) falls to 70.7% of its maximum value. This corresponds to a reduction of power by a factor of two.

$$\Delta f = f_1 - f_2 \quad (5)$$

The quality factor Q is defined as the ratio of the resonance frequency over the width:

$$Q = f_o / \Delta f \quad (6)$$

The quality factor Q is a measure of the tuning quality of a resonance circuit. The higher the quality factor, the smaller the band width Δf, and therefore the less interference between the adjacent channels.

Procedure:

1. Construct a circuit shown in Fig. (1). Measure and record the resistance R. You will need the resistance to calculate the current.

2. Read the capacitance C and the inductance L from the devices. Record these parameters. Calculate the theoretical resonance frequency f_o using Eq. (4). This expected resonance frequency may not necessarily exactly equal to the experimental resonance frequency due to the experimental errors. But the expected value does give your a guidance as to what frequency range you should be working in to get the entire resonance curve.

3. With the guidance from step 2, set the signal generator in the range covering the expected resonance frequency. Measure the total voltage across the capacitor and the inductor, V_{LC}, with the oscilloscope. The amplitude of this voltage should change while you are changing the frequency of the generator. Change the frequency slowly while watching the change of the amplitude of V_{LC} until you **find the frequency at which the voltage V_{LC} is the minimum.** This frequency is the experimental value of the resonance frequency. But do not trust the frequency reading of the front panel of the signal generator. Use the oscilloscope to measure the period of the waveform and calculator the frequency which is the reciprocal of the period. The oscilloscope measurement is more reliable than the frequency value read from the signal generator.

4. Having obtained the experimental resonance frequency, you can then map out the resonance curve. You need to obtain about half dozen points of each side of the resonance peak. For each frequency point, you need to measure both the resonance voltage V_{LC} and the current I. The current I can be obtained by measuring the voltage across the resistor, again using the oscilloscope, divided by the resistance measured at the beginning of the experiment.

Data analysis:

1. Plot the ratio (I/V_{LC}) versus the frequency f. You should get a resonance curve as shown in Fig. 2.
2. Use the method shown in Fig. 2 to obtain the band width Δf and the quality factor Q.