ACCELERATION DUE TO GRAVITY

Objective: To measure the acceleration of a freely falling body due to gravitational attraction.

Apparatus: Computer with Logger Pro, green Vernier interface box, picket fence with photogate.

(Left) Picket Fence. (Right) Photogate.

Theory:

An object's motion at any point in time, t, can be described by its position, x, (with respect to a reference starting point, x_0); its velocity, v, and its acceleration, a. These are given by:

$$v = \frac{\text{change in position}}{\text{change in time}} = \frac{(x - x_0)}{(t - t_0)} \quad \textbf{\textit{(eq. 1)}} \qquad \text{and} \qquad a = \frac{\text{change in velocity}}{\text{change in time}} = \frac{(v - v_0)}{(t - t_0)} \quad \textbf{\textit{(eq. 2)}}$$

where t_0 is the time that the clock started (usually assumed to be 0 seconds).

Solving for the change in position (or distance, *d*, travelled) from eq. 1 yields $d=vt$. Furthermore, for an object undergoing a constant acceleration, the distance traveled would be equal to the average velocity (half-way between the velocities at 2 endpoints, $v = ½ (v + v_0)$), times the time:

$$d = \frac{1}{2}(v_0 + v)t \qquad \textbf{\textit{(eq.3)}}$$

A further expression for the distance an object has travelled in a given time if it is undergoing a constant acceleration can be derived by solving for velocity in eq. 2, which yields:

$$v = v_0 + at \qquad \textbf{\textit{(eq. 4)}}.$$

Combining the expressions in equations 3 and 4 gives:

$$d = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(v_0 + v_0 + at)t = \frac{1}{2}(2v_0 t) + \frac{1}{2}a(t)(t) = v_0 t + \frac{1}{2}at^2 \qquad \textbf{\textit{(eq.5)}}$$

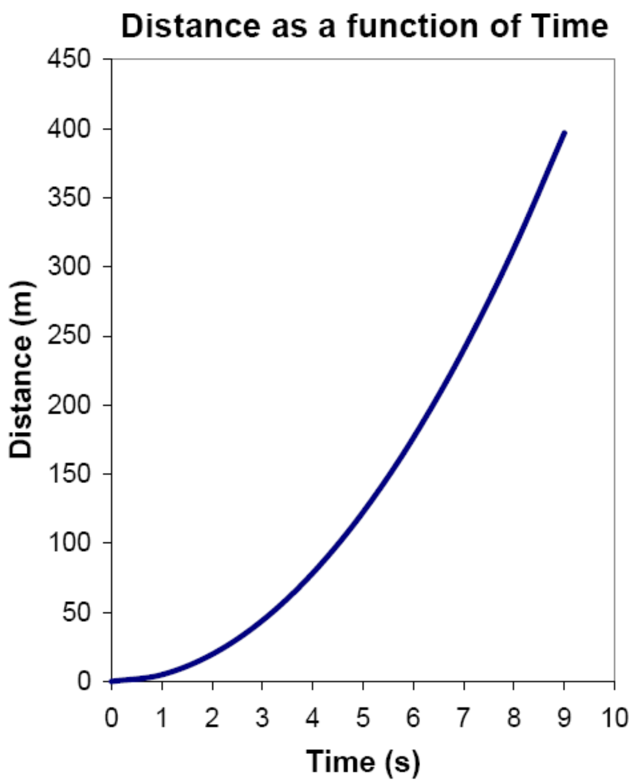
For an object in free-fall motion, the motion is only in one direction (down), and the acceleration is the acceleration due to gravity. Acceleration due to gravity is then the rate at which the velocity of a freely falling body will change under the influence of the gravitational force alone. For a falling body starting from an origin at d_0, the equations then become:

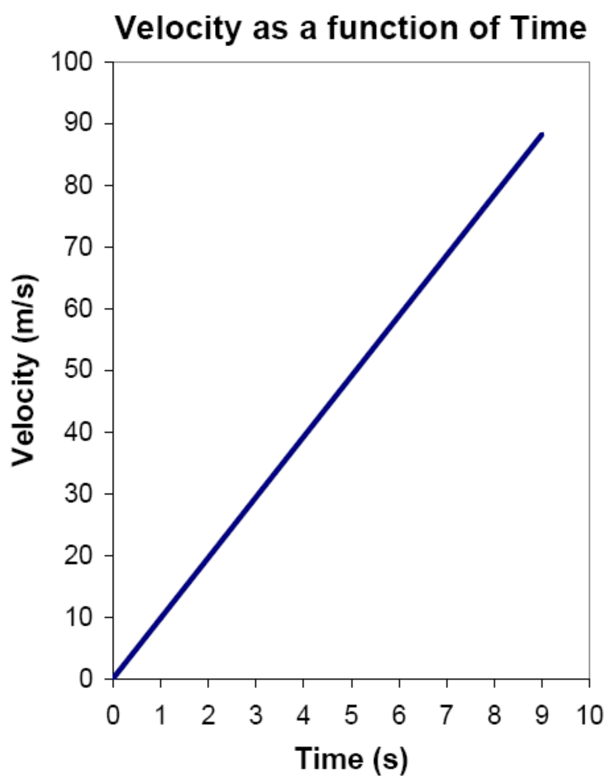
$$a = g \text{ (constant)}; \quad v = v_0 + gt; \quad \text{and} \quad d = d_0 + v_0 t + ½\, gt^2. \quad \textbf{\textit{(eq. 6, 7, and 8)}}$$

where g is the average acceleration due to gravity on earth, for which the accepted value is 9.81 m/s^2.

In this experiment, we will be obtaining a value for the acceleration due to gravity by measuring the distance that a falling object travels as a function of time. The equipment provided includes a computer controlled photogate and a “picket fence”. The photogate is an instrument that senses if an object that absorbs light is located between the two arms of the instrument. Essentially, one arm has a light bulb that’s presence is detected by a photodiode located in the other arm. If something blocks the flow of light, the photogate alerts the computer controller. The picket fence is simply a strip of alternating dark-and-clear plastic. If one, for example, drops the picket fence between the arms of the photogate, then the photogate would alternately report to the computer either that the gate was blocked or unblocked, depending on how far the picket fence had fallen. Because the computer is pre-programmed to know the width of the alternating light and dark sections of the picket fence (and hence the relative position of the object), the software can generate a set of data for position as a function of time for the picket fence.

According to equations 7 and 8, for a freely-falling object that starts from rest at t = 0 s, the distance traveled and velocity at t = 1 s will be 4.9 m and 9.8 m/s, respectively. At t = 2 seconds, the object will have travelled 19.6 m, and its velocity will be 19.6 m/s. If the experiment were continued and air resistance ignored, we find that plots of distance vs. time and velocity vs. time will look like:





Procedure:

Before you begin, please note and take extra caution: You will be releasing the plastic picket fences from rest. Notice that the floor of the lab is concrete. The picket fences will break if they are dropped from the height of the table to the floor. To prevent this, place the cushioning material provided to you directly under the photogate where the picket fence will land.

1. Clamp a photogate to a ring stand and position it so that it hangs off the side of the table. Make sure the picket fence can fall through the photogate without touching the photogate. Position the foam padded box under the photogate for the picket fence to fall into. Practice dropping the picket fence into the box a few times before collecting the data you will use for your analysis.

Experimental set-up

2. Prepare your computer for data collection. Open the Logger Pro program. Within that progam, open the Physics with Vernier folder, then “05 Picket Fence Free Fall.cmbl”. This brings up a dialog that will help you connect the photogate appropriately to the green interface that is connected to your computer. Plug in the photogate to the port indicated by this dialog box.

3. To start the experiment, click on the “Collect” icon. Position the picket fence just above the photogate and hold it as plumb as possible so that all of the black strips on the fence will pass through the gate completely vertically. Be careful not to activate the sensors before you drop the picket fence. Drop it through the photogate. If it was not done automatically, now press the “Stop” button located where the “Collect” button previously appeared. There should be data points in both windows (the velocity vs. time and the distance vs. time graphs). Click on the A at the top of the window to autoscale your graphs.

4. Ask the computer to analyze the data for the velocity vs. time graph. Click on this graph, click the “Analyze” menu option, and choose “Curve fit”. You should fit this data with a linear function (mt+b). Then click on “Try fit” and OK. As a result of the analysis you will see a box with an equation of format ***y=mx+b***. Record the values for ***m*** and ***b*** on your data page.

5. Analyze the data for the distance vs. time graph by clicking on the graph, going to the Analyze menu, then "Curve Fit", and choosing the equation for a polynomial that looks similar to a parabola (the general form is $\boldsymbol{Ax^2+Bx+C}$). Record the values of *A*, *B*, and *C* on your data page.

6. Print a copy of the distance vs. time graph and the velocity vs. time graph for your report. You will need only 1 set of graphs per lab partner for your report.

7. Repeat steps 3 through 5 five more times.

Data Analysis:

1. Determine the proper units for each parameter from the data fit and record them in the appropriate row of the data table on the data sheet.

2. For the data from the graphs of distance vs. time, find the average and standard deviation of the parameters ***A***, ***B***, and ***C***. Your results should be reported in the form $A_{avg} \pm \sigma_A$; $B_{avg} \pm \sigma_B$; $C_{avg} \pm \sigma_C$. Comparing the equation $y=Ax^2 +Bx+ C$ with the equation $d = d_0 + v_0t + ½ gt^2$, you should see that *A* corresponds to the value for $½ g$, and *B* corresponds to the value for v_0. Calculate your value for *g* from this data.

3. For the data from the graphs of velocity vs. time, find the average and standard deviations for ***m*** and ***b*** using your data. Report your results in the form $m = m_{avg} \pm \sigma_m$; $b = b_{avg} \pm \sigma_b$. Comparing the theoretical equation $v = v_0 + gt$ to the equation for a line, $y = mx + b$, you should find that the slope, m, corresponds to the acceleration due to gravity, and the value of b corresponds to the initial velocity, v_0.

4. Calculate the relative percent deviation between your value for m_{avg} and the accepted value of 9.81 m/s^2 for *g*.

Discussion Questions:

1. The constants in the mathematical expressions used to fit both sets of data have a physical meaning. What is the physical meaning of C in the expression used to fit the distance vs. time data? What does the value you determined for C tell you about your specific experiment? Also, from the fit on the graph of velocity vs. time, the y-intercept is usually non-zero. Explain this result in context of what was happening in your experiment.

2. Since the description of motion for an object (i.e., position, velocity, and acceleration) are all related to the variable of time, the shapes of the curves on a plot of each kinematic variable are related to one another. For example, the slope of the distance vs. time graph is the velocity, and the slope of the velocity vs. time graph is the acceleration of the object. What would a plot of the acceleration of the picket fence with respect to time look like?

3. The accepted average value for *g* anywhere on Earth is 9.81 m/s^2. This is an average value, and local fluctuations due to latitude, altitude, the position of the Sun relative to the location on Earth, etc. can account for minor differences (<1%) from the accepted average value.

Elsewhere in the universe, the gravitational pull of a body has different values, such as on the Moon, where the acceleration due to gravity is approximately 1/6 of the value on Earth. If you did this exact experiment on the moon, what would your data look like (consider the shape of the fits and parameters that describe them)?

Lab Report Format:

Your lab report for this experiment should contain:

1. Pre-lab (objective, theory, sketch of the experimental setup, and procedure).
2. Neatly written experimental data page.
3. Sample calculations: For this lab, an example needs to be shown for calculation of the average of one of the parameters and its standard deviation (choose either *A*, *B*, *C*, *m*, or *b*), the percent deviation of m_{avg} from the accepted value of *g*, and the calculation of *g* from the fit to the graph of distance vs. time.
4. Graphs: Include with your data sheet two graphs, one of velocity vs. time and one of distance vs. time for one of the data trials.
5. Results: State your results in the form of complete sentences. For both sets of graphs, you should report the shape of the curve and the mathematical function you fit it to, along with the results of the fitting parameters for A, B, C, m, and b (all rounded properly and in the form avg $\pm$ σ). Explain what each of these parameters physically represents. Compare your values for *B* and *b*. Are they within the uncertainty range of each other? Compare the values of g obtained from both graphs. Are they within the uncertainty range of each other? Finally, report the relative percent deviation of m_{avg} from the accepted value of g. Is the agreement good?
6. Conclusions: Analyze the method used to determine *g* in this experiment, addressing the answers to the three discussion questions above.