CONSERVATION OF MOMENTUM

Objective: To determine if momentum is conserved in a two-dimensional collision between two objects.

Apparatus: Conductive paper, newsprint, frictionless gliders, compressed air, spark timer, meter stick, a protractor and ruler.

Theory:

The momentum, p, that an object possesses is defined by the vector equation:

$$p = m\mathbf{v} \qquad \textbf{(eq. 1)}$$

where m is the mass of the object and **v** is its velocity. Newton's 2nd law of motion, F = ma, can also be written in terms of momentum:

$$\mathbf{F} = m\mathbf{a} = m\frac{\Delta\mathbf{v}}{\Delta t} = \frac{\Delta\mathbf{p}}{\Delta t} \qquad \textbf{(eq. 2)}$$

If there is no net or externally applied force acting on the object (F = 0), then

$$\mathbf{F} = \frac{\Delta\mathbf{p}}{\Delta t} = 0 \quad \text{or} \quad \Delta\mathbf{p} = 0 \qquad \textbf{(eq. 3)}$$

In the case of a collision between two objects, m_A and m_B, they exert equal and opposite forces on each other, called internal forces. If the only forces involved are internal forces and no externally applied force intervenes (F = 0), the total momentum of the colliding bodies is said to be conserved, (Δp = 0). That is:

$$\underbrace{\mathbf{p}_{A1} + \mathbf{p}_{B1}}_{\text{(before)}} = \underbrace{\mathbf{p}_{A2} + \mathbf{p}_{B2}}_{\text{(after)}} \qquad \textbf{(eq. 4)}$$

or

$$m_A\mathbf{v}_{A1} + m_B\mathbf{v}_{B1} = m_A\mathbf{v}_{A2} + m_B\mathbf{v}_{B2} \qquad \textbf{(eq. 5)}$$

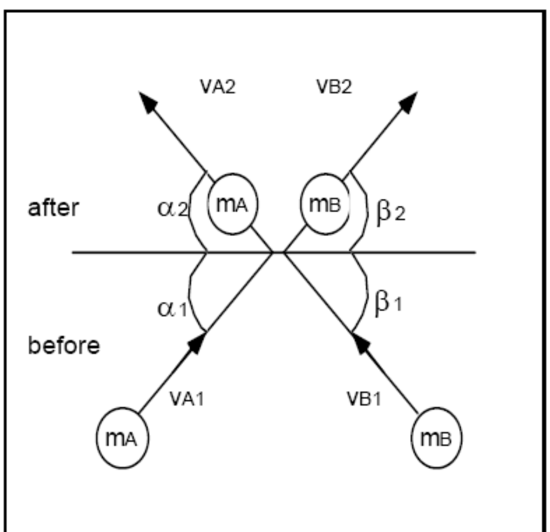


Figure 1

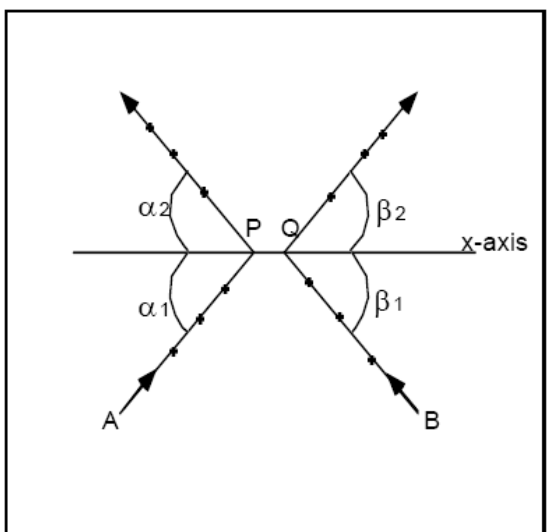


Figure 2

Procedure and Data Analysis:

PART I. To record a collision

1. Tape the conductive paper to the top of the lab table and place the newsprint over it.
2. Turn on the air compressor so that the two gliders rise on a cushion of air above the newsprint.
3. Practice colliding the gliders in the middle of the newsprint until you can cause a collision with only a small impulse, or applied force, to the gliders. Each mass should be aimed for the center of the paper from different angles and with a different impulse.
4. Connect the spark timer to the gliders and conductive paper, turn on the timer, make a collision, and turn off the timer as the gliders reach the edge of the paper. The sparks between the conductive paper and the gliders mark the bottom of the newsprint at the frequency indicated on the spark timer (usually 60 Hz). If the spark timer has a frequency of 60 Hz, then the position is marked every 1/60 of a second.
5. Remove the newsprint and label the start and finish of the A track and B track.

PART II. To find the magnitudes of the velocities of m_A and m_B before and after collision.

1. Draw straight line segments through the dots (four lines), such as shown in Figure 2. The two points of intersection correspond to the gliders' centers at the time of the collision. Discard the dots which were made while the gliders were being pushed (where an external force was being applied).
2. In each track segment, measure the distance the gliders traveled during the time t. For the measurement of each distance, first pick a dot closest to the collision point to be the zero position on each track segment. The endpoint for the distance should be as far away from the zero point as possible without getting to the range where external forces may be applied to the masses. If the frequency of the timer was 60 Hz, this is usually between the range of 1/6 second (10 time intervals, or 11 dots) and ½ second (30 time intervals, or 31 dots). Record the four distances on your data sheet as x_{A1}, x_{A2}, x_{B1}, and x_{B2}. The subscript 1 and 2 refer to before and after the collision, respectively, and the subscripts A and B correspond to each glider.
3. Do not forget to record the uncertainty, Δx, associated with these distance measurements. These will be ½ of the smallest division on your measuring scale. (Example: on a ruler with 1 mm as the smallest division, the error is 0.5 mm or 0.0005 m).
4. Record the time interval that corresponds to each measured distance on your data sheet, t_{A1}, t_{A2}, t_{B1}, and t_{B2}. We will assume that there is no uncertainty in the accuracy of the timing device; however, as a good rule of thumb, we should not keep digits beyond the fourth decimal place, since that is how many digits we can estimate on the distance measurement in meters (where the last digit is always a 0 or 5).

5. Calculate the velocities of each track, using the definition of velocity as distance divided by time, or $v = \frac{x}{t}$. Each uncertainty in velocity, Δv, will be found by dividing Δx by its associated t, or $\Delta v = \frac{\Delta x}{t}$. Record these on your data sheet.

PART III. To find the magnitudes of the momenta p_{A1}, p_{B1} and p_{A2}, p_{B2}.

1. Record the values for the masses of gliders A and B, and their associated uncertainties, Δm. These are typically 0.210 kg and 0.555 kg. Take Δm to be 5% of the total mass, m, which is the uncertainty due to the attached rubber tubing.

2. In order to find the magnitude of the momentum for each track (p_{A1}, p_{B1} and p_{A2}, p_{B2}), multiply the velocities by the masses for each track (p = mv) and record this on your data sheet. The standard units of momentum are kg·m/s.

3. Calculate the uncertainty in the magnitude of the momentum for each track, Δp_{A1}, Δp_{B1} and Δp_{A2}, Δp_{B2}. By propagation of error techniques, it can be shown that

$$\Delta p = mv\left(\frac{\Delta m}{m} + \frac{\Delta v}{v}\right).$$

   Record these on your data sheet.

4. Calculate the magnitude of the total momentum before the collision and its associated error, and the magnitude of the total momentum after the collision with its associated error. The magnitude of the total momentum before the collision will be $P_1 = P_{A1} + P_{B1}$. The uncertainty in the magnitude of the momentums before the collision will be $\Delta P_1 = \Delta P_{A1} + \Delta P_{B1}$ (as errors always add). Do the analogous calculations for after the collision to find $P_2 \pm \Delta P_2$. Record these values on your datasheet.

PART IV. To construct a vector diagram to indicate the directions of the momenta vectors.

We have now calculated the magnitudes of the momenta, but in order to determine whether momentum is conserved, we must calculate their directions, as momentum is a vector. To do this, we will construct a vector diagram, such as shown in Figure 3 (below).

1. Calculate the lengths of each momentum vector by using an appropriate scale, such that your diagram will be able to fit on an 8.5 x 11 inch sheet of paper to include with your lab report. For example: A momentum p was 0.34 ± 0.02 kg m/s. Using a scale of 2.5 cm for every 0.100 kg m/s, the length of this momentum vector will be 8.5 ± 0.5 cm. Record the scaling factors and the scaled lengths of the momenta on your data sheet.

2. There are two points of intersection of straight line segments through the dots that correspond to the gliders' centers at the time of the collision (shown in figure 2). Draw another line (which we will define as the x-axis) through these two points of intersection.

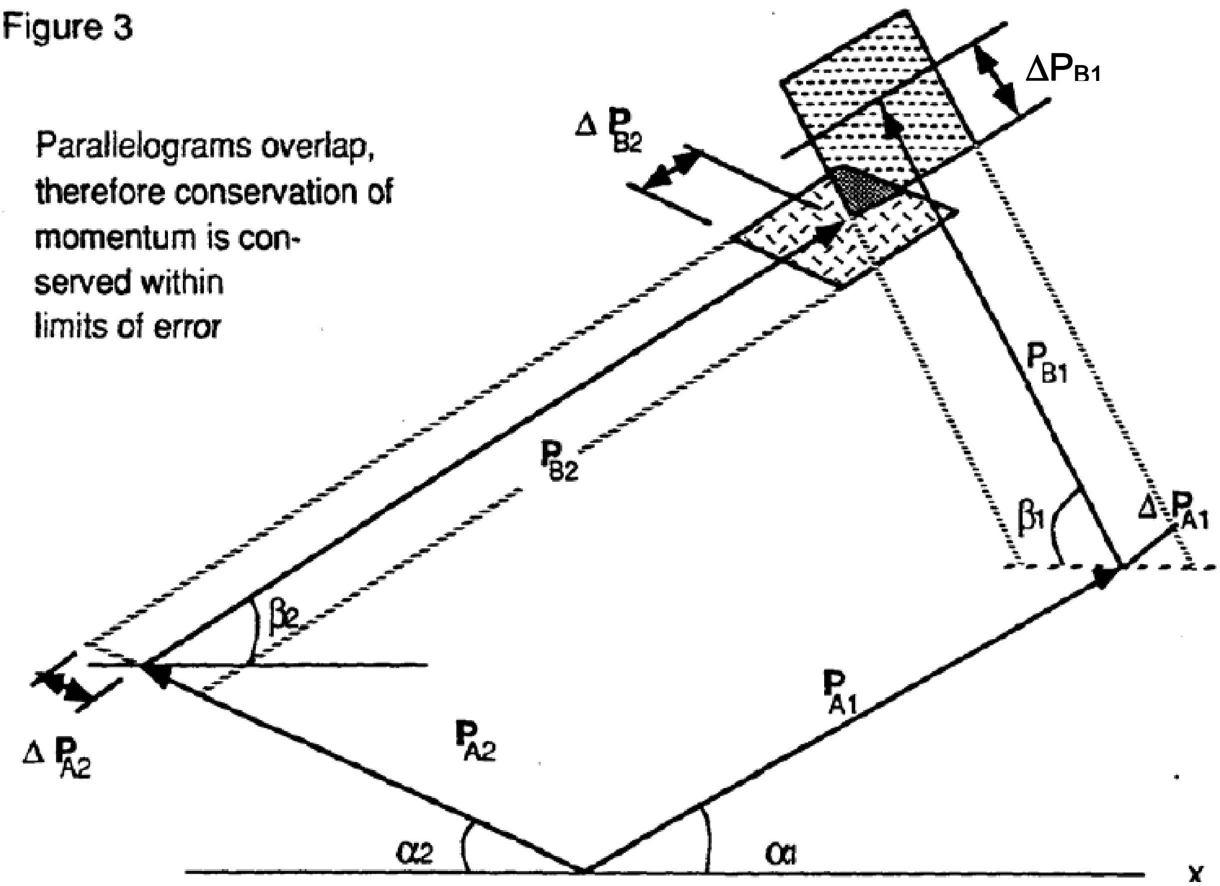
3. Measure and record the angle that each track makes with the x-axis, α_1, α_2, β_1, and β_2 (shown in figure 2).

4. On a new sheet of paper, draw a horizontal line, which will be the x-axis (shown at the bottom of figure 3). At a new origin in the center of this x-axis, draw vector p_{A1} the length of the scaled magnitude that was calculated for it, and at the direction of the angle measured, α_1.

5. From the head of vector p_{A1}, make another x-axis in the same direction as the original x-axis and then draw vector p_{B1}, to the scaled length and direction β_1, as shown in figure 3.

6. At the same origin as in step 4 above, repeat steps 4 and 5 above for the scaled magnitudes of p_{A2} and p_{B2} with their associated angles, α_2 and β_2, respectively, as shown in figure 3 below.

Figure 3



PART V. To draw the uncertainties in the momentum vectors in order to generate a region which represents a range of possible values for the total momentum. (Shown in figure 3, above).

1. On the vector diagram, extend the vector p_{A1} by the scaled length of uncertainty, Δp_{A1}, (calculated in part IV, step 1) forward and backward from the head of p_{A1}. This range will represent the locations where it is possible that the head of the vector p_{A1} may have been.

2. Draw two lines parallel to p_{B1} that are the distance Δp_{A1} apart, as shown in figure 3.

3. Extend the vector p_{B1} by the scaled length of uncertainty Δp_{B1} (calculated in part IV, step 1) both forward and backward from the head of vector p_{B1}.

4. From the head of p_{B1}, draw two lines that are parallel to p_{A1} that are the distance Δp_{B1} apart, as shown in figure 3. This should form a parallelogram of uncertainty at the head of p_{B1}.

5. Repeat steps 1-4 above for p_{A2} and p_{B2}.

6. The parallelograms of uncertainty should overlap, which would indicate that momentum is conserved within the limits of this experiment.

Discussion Questions:

1. Why is it necessary in this experiment that the masses are "gliding" on a cushion of air? What part of our theoretical assumptions in the design of this experiment would be invalid if the masses were not on an air cushion?

2. If the mass of both glider A and glider B were doubled, but were given the same initial impulse (magnitude and direction) to set them in motion, how would their before and after collision velocities change?

3. There are two types of collisions: elastic and inelastic. In an elastic collision, the colliding objects rebound, with no energy lost to deformation of the objects or as heat. In an inelastic collision, the colliding objects might stick together, be deformed in some way, or lose energy due to friction as heat. For both types of collisions, the momentum of the system is conserved. Kinetic energy of the total system is only conserved in an elastic collision; it is not conserved for an inelastic collision.

   Which type of collision between the gliders did our experiment involve? Support your conclusion with a calculation of the kinetic energies of the gliders before and after the collisions, where $KE = \frac{1}{2}mv^2$ (in units of Joules if m is in kg and v is in m/s). If KE is conserved, then $KE_{before} = KE_{after}$, or

   $$\frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2 \quad .$$

   Is there a large difference between the kinetic energies before and after the collision? (You can do a relative % difference calculation to show this):

   $$\%\,\text{difference} = \left(\frac{\text{KE}_{\text{before}} - KE_{after}}{\text{KE}_{\text{before}}}\right) \times 100\% \,.$$

Since collisions can be completely elastic, completely inelastic or somewhere in between, calculate the errors in the KE before and after the collision to see if the KE's before and after the collisions overlap.

$$\Delta KE = \frac{1}{2} m v^2 \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(2\,\frac{\Delta v}{v}\right)^2}$$

Lab Report Format:

Your lab report for this experiment should contain:

1. Pre-lab (objective, theory, sketch of the experimental design, and procedure).
2. Neatly written experimental data. This includes both (a) the data sheet and (b) your scaled vector diagram which must be handed in with the report.
3. Sample calculations: For this lab, you need to show (a) a sample calculation for one of the velocities and its uncertainty, e.g. $v_{A1} \pm \Delta v_{A1}$; (b) a sample calculation of one of the magnitudes of the momentum and its uncertainty, e.g. $p_{A1} \pm \Delta p_{A1}$; (c) a sample calculation for the scaled magnitude for one of the momenta magnitudes and its uncertainty; (d) a sample calculation for the magnitude of the total momentum before the collision and its associated un certainty, $P_1 \pm \Delta P_1$; (e) drawing for parallelograms of uncertainty on your vector diagram indicating the uncertainty in both the magnitude and the direction of the momentum vectors; (f) calculations of the kinetic energies, the errors in the KE's (ΔKE) and the % difference as needed to answer discussion question 3.
4. Results: Report the values for the magnitudes of the total momentum before and after the collision, $P_1 \pm \Delta P_1$ and $P_2 \pm \Delta P_2$. Make sure all values are properly rounded, have the correct number of significant digits, and are expressed with proper units. Are these values within the uncertainty range of one another? When you consider the direction of the momentum vectors, how large is your uncertainty (indicated by the area of overlapping parallelograms)? Was the momentum conserved within the error range of the experiment?
5. Conclusions: The conclusions to a published scientific study generally usually discuss sources of error in experiment design, predict results of other experiments based on the results of the current one, and discuss other aspects of the system studied.

    Discuss the design of this experiment by answering discussion question 1. Discuss momentum and its conservation in this system by using the results of this experiment to predict the results to a different one (i.e., answer discussion question 2). Discuss the type of collision and whether conservation of kinetic energy is also verified in this experiment with discussion question 3.