8-3 Escape Speed

**Vocabulary**

**Escape Speed:** The minimum speed an object must possess in order to escape from the gravitational pull of a body.

In Chapter 6, you worked with gravitational potential energy and kinetic energy. When an object moves away from Earth, its gravitational potential energy increases. Since its total energy is conserved, its kinetic energy decreases. When the object is close to Earth, the gravitational force on it is a fairly constant \( mg \). However, as you know, the gravitational force drops rapidly as you get farther from Earth. If an object moves upward from Earth with enough speed, it will never run out of kinetic energy and will escape from Earth.

The escape speed for an object leaving the surface of any celestial body of mass \( M \) and radius \( d \) is

\[
v = \sqrt{\frac{2GM}{d}}
\]

Notice that the mass of the escaping object does not affect the escape speed.

**Solved Examples**

**Example 6:** Earth has a mass of \( 5.98 \times 10^{24} \) kg and a radius of \( 6.38 \times 10^6 \) km. What is the escape speed of a rocket launched on Earth?

*Given:* \( M = 5.98 \times 10^{24} \) kg  
\( d = 6.38 \times 10^6 \) m  
\( G = 6.67 \times 10^{-11} \) N·m²/kg²

*Unknown:* \( v = ? \)  
*Original equation:* \( v = \sqrt{\frac{2GM}{d}} \)

*Solve:* \( v = \sqrt{\frac{2GM}{d}} = \sqrt{\frac{2(6.67 \times 10^{-11} \text{ N·m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{6.38 \times 10^6 \text{ m}}} \)

\[= 11,200 \text{ m/s}\]

Any rocket trying to escape Earth’s gravitational pull must be going at least 11,200 m/s before engine cut-off, in order to get away.

**Example 7:** Compare Example 6 with the escape speed of a rocket launched from the moon. The mass of the moon is \( 7.35 \times 10^{22} \) kg and the radius is \( 1.74 \times 10^6 \) m.

*Given:* \( M = 7.35 \times 10^{22} \) kg  
\( d = 1.74 \times 10^6 \) m  
\( G = 6.67 \times 10^{-11} \) N·m²/kg²

*Unknown:* \( v = ? \)  
*Original equation:* \( v = \sqrt{\frac{2GM}{d}} \)

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Solve: \[ v = \sqrt{\frac{2GM}{d}} = \sqrt{\frac{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(7.35 \times 10^{22} \text{ kg})}{1.74 \times 10^6 \text{ m}} = 2370 \text{ m/s} \]

Notice that you can escape from the moon by traveling much more slowly than you must travel to escape the gravitational pull of Earth. This is why launching a Lunar Module from the moon’s surface was so much easier than launching an Apollo spacecraft from Earth.

**Practice Exercise**

**Exercise 11:** How fast would you need to travel a) to escape the gravitational pull of the sun? \((M_S = 1.99 \times 10^{30} \text{ kg}, d_S = 6.96 \times 10^8 \text{ m})\) b) As the sun begins to die, it will become a red giant. This means that its mass will remain the same but its diameter will increase substantially (perhaps even out as far as Earth’s orbit!). When the sun becomes a red giant, will its escape speed be greater than, less than, or the same as it is now?

**Answer:** a. ______________________

**Answer:** b. ______________________

**Exercise 12:** How fast would the moon need to travel in order to escape the gravitational pull of Earth, if Earth has a mass of \(5.98 \times 10^{24} \text{ kg}\) and the distance from Earth to the moon is \(3.84 \times 10^8 \text{ m}\)?

**Answer:** ______________________
Exercise 13: What is the escape speed needed a) to escape the gravitational pull of Asteroid B612 (see Exercise 7)? b) What would happen if you jumped up on Asteroid B612?

Answer: a. ______________________

Answer: b. ______________________

Exercise 14: Scotty finds it difficult to play catch on planet Apgar because the planet’s escape speed is only 5.00 m/s, and if Scotty throws the ball too hard, it flies away. If planet Apgar has a mass of $1.56 \times 10^{15}$ kg, what is its radius?

Answer: ______________________

Additional Exercises

A-1: Halley’s Comet orbits the sun about every 75 years due to the gravitational force the sun provides. Compare the gravitational force between Halley’s Comet and the sun when the comet is at aphelion (its greatest distance from the sun) and $d$ is about $4.5 \times 10^{12} \text{ m}$ to the force at perihelion (or closest approach), where $d$ is about $5.0 \times 10^{10} \text{ m}$.

A-2: In Exercise A-1, what is the comet’s acceleration a) at aphelion? b) at perihelion? ($M_S = 1.99 \times 10^{30} \text{ kg}$)
A-3: An early planetary model of the hydrogen atom consisted of a $1.67 \times 10^{-27}$-kg proton in the nucleus and a $9.11 \times 10^{-31}$-kg electron in orbit around it at a distance of $5.0 \times 10^{-11}$ m. In this model, what is the gravitational force between a proton and an electron?

A-4: At what height above Earth would a 400.0-kg weather satellite have to orbit in order to experience a gravitational force half as strong as that on the surface of Earth?

A-5: It is said that people often behave in unusual ways during a full moon.
   a) Calculate the gravitational force that the moon would exert on a 50.0-kg student in your physics class. The moon is $3.84 \times 10^8$ m from Earth and has a mass of $7.35 \times 10^{22}$ kg. b) Does the moon attract the student with a force that is greater than, less than, or the same as the force with which the student attracts the moon?

A-6: The tiny planet Mercury has a radius of 2400 km and a mass of $3.3 \times 10^{23}$ kg.
   a) What would be the gravitational acceleration of an astronaut standing on the surface of Mercury? b) Compare the motion of a ball dropped on the surface of Mercury to that of a ball dropped on Earth.

A-7: The acceleration due to gravity on Venus is 0.89 that of Earth.
   a) If the radius of Venus is $6.05 \times 10^6$ m, what is Venus’ mass? b) How does this compare to Earth’s mass? c) If you were on a diet and had to “weigh in,” would you rather stand on a scale on Venus or on Earth in order to appear as if you had lost the most weight?

A-8: The planet Mars has a mass that is 0.11 times Earth’s mass and a radius that is 0.54 times Earth’s radius.
   a) How much would a 60.0-kg astronaut weigh if she were to stand on the surface of Mars? b) Although Mercury is much smaller than Mars, it has almost the same gravitational acceleration. Describe how you might explain this phenomenon.

A-9: NASA’s Dawn spacecraft is scheduled to reach the asteroid Ceres in February of 2015. Ceres is the largest asteroid ever discovered and orbits in the asteroid belt between Mars and Jupiter. Ceres has a mass of $9.46 \times 10^{20}$ kg and a radius of about $4.25 \times 10^5$ m. What gravitational acceleration would Ceres provide on the spacecraft if it landed on the surface of the asteroid? (Read more about the Dawn spacecraft at http://dawn.jpl.nasa.gov)

A-10: Find the Dawn spacecraft’s escape speed from Ceres, using the information given in A-9.

A-11: On August 4, 2007, NASA’s Phoenix Mars mission blasted off on its way to the red planet to test rock and ice samples for signs of life. If the mass of the Phoenix is 350 kg, a) what would be the weight of the spacecraft on the planet’s surface and b) what escape speed would be needed in order for a spacecraft of this size to be launched off the surface of Mars? ($M_m = 6.42 \times 10^{23}$ kg, $d_M = 3.39 \times 10^8$ m) (Read more about this mission to Mars at http://www.nasa.gov/mission_pages/phoenix/main/index.html)
Challenge Exercises for Further Study

B-1: At what distance from Earth’s center must a spacecraft be in order to experience the same gravitational attraction from both Earth and the moon when directly between the two? \(M_E = 5.98 \times 10^{24} \text{ kg}, M_M = 7.35 \times 10^{22} \text{ kg}, d_{E-M} = 3.84 \times 10^8 \text{ m}\)

B-2: Jupiter’s innermost Galilean satellite, Io, is covered with active volcanoes, which exist because of the immense gravitational tugging on the satellite by Jupiter and the other moons near Io. Io orbits \(4.2 \times 10^8 \text{ m}\) from the center of Jupiter. The other Galilean satellites are located as follows from Jupiter’s center: Europa: \(6.7 \times 10^8 \text{ m}\), Ganymede: \(1.0 \times 10^9 \text{ m}\), and Callisto: \(1.9 \times 10^9 \text{ m}\). If Jupiter and its satellites are lined up as shown, what gravitational force does the satellite Io experience? \(M_I = 8.9 \times 10^{22} \text{ kg}, M_E = 4.9 \times 10^{22} \text{ kg}, M_G = 1.5 \times 10^{24} \text{ kg}, M_C = 1.1 \times 10^{23} \text{ kg}, M_J = 1.9 \times 10^{27} \text{ kg}\)

B-3: Saturn’s satellite, Titan, orbits the planet in a little less than 16 days. Titan orbits Saturn at an average distance of \(1.216 \times 10^9 \text{ m}\) from the center of the planet. Use this information to find the mass of Saturn.